6.2 Factoring Trinomials

Objectives

1 Factor trinomials when the coefficient of the quadratic term is 1.

2 Factor trinomials when the coefficient of the quadratic term is not 1.

3 Use an alternative method for factoring trinomials.

4 Factor by substitution.

PEARSON

Factor trinomials when the coefficient of the quadratic term is 1.

Factoring $x^2 + bx + c$

Step 1 **Find pairs whose product is *c*.** Find all pairs of integers whose product is c, the third term of the trinomial.

Step 2 **Find pairs whose sum is *b*.** Choose the pair whose sum is b, the coefficient of the middle term.

If there are no such integers, the polynomials cannot be factored.

A polynomial that cannot be factored with integer coefficient is a **prime polynomial**.

CLASSROOM EXAMPLE 1 **Factoring Trinomials in $x^2 + bx + c$ Form**

Factor the trinomial.

$a^2 + 9a + 20$

Solution:

Step 1 Find pairs of numbers whose product is 20.	***Step 2*** Write sums of those numbers.
$20(1)$	$20 + 1 = 21$
$-20(-1)$	$-20 + (-1) = -21$
$10(2)$	$10 + 2 = 12$
$-10(-2)$	$-10 + (-2) = -12$
$5(4)$	$5 + 4 = 9$
$-5(-4)$	$-5 + (-4) = -9$

CLASSROOM EXAMPLE 1 **Factoring Trinomials in $x^2 + bx + c$ Form (cont'd)**

The coefficient of the middle term is 9, so the required numbers are 5 and 4. The factored form of $a^2 + 9a + 20$ is

$$(a + 5)(a + 4).$$

Check $(a + 5)(a + 4) = a^2 + 9a + 20$

CLASSROOM EXAMPLE 1 **Factoring Trinomials in $x^2 + bx + c$ Form (cont'd)**

Factor the trinomial.

$b^2 - 7b + 10$

Solution:

Step 1 Find pairs of numbers whose product is 10.	***Step 2*** Write sums of those numbers.
$10(1)$	$10 + 1 = 11$
$-10(-1)$	$-10 + (-1) = -11$
$5(2)$	$5 + 2 = 7$
$-5(-2)$	$-5 + (-2) = -7$

CLASSROOM EXAMPLE 1 **Factoring Trinomials in $x^2 + bx + c$ Form (cont'd)**

The coefficient of the middle term is –7, so the required numbers are –5 and –2. The factored form of $b^2 - 7b + 10$ is

$$(b - 5)(b - 2).$$

Check $(b - 5)(b - 2) = b^2 - 7b + 10$

CLASSROOM EXAMPLE 2 **Recognizing a Prime Polynomial**

Factor $t^2 + 3t – 5$.

Solution:

Look for two expressions whose product is –5 and whose sum is 3. There are no such quantities. Therefore, the trinomial cannot be factored and is ***prime.***

CLASSROOM EXAMPLE 3 **Factoring a Trinomial in Two Variables**

Factor $p^2 – 5pq + 6q^2$.

Solution:

Look for two expressions whose product is $6q^2$ and whose sum is –5q. The quantities –3q and –2q have the necessary product and sum, so

$$p^2 – 5pq + 6q^2 = (p – 3q)(p – 2q).$$

CLASSROOM EXAMPLE 4 **Factoring a Trinomial with a Common Factor**

Factor $3a^3 + 12a^2 – 15a$.

Solution:

Start by factoring out the GCF, 3a.

$$= 3a(a^2 + 4a – 5)$$

To factor $a^2 + 4a – 5$, look for two integers whose product is –5 and whose sum is 4. The necessary integers are –1 and 5. Remember to write the common factor 3a as part of the answer.

$$= 3a(a – 1)(a + 5)$$

CAUTION ***When factoring, always look for a common factor first. Remember to write the common factor as part of the answer.***

Objective 2

Factor trinomials when the coefficient of the quadratic term is not 1.

CLASSROOM EXAMPLE 5 **Factoring a Trinomial in $ax^2 + bx + c$ Form**

Factor $6k^2 – 19k + 10$.

Solution:

The product as is 6 (10) = 60. Look for two integers whose products is 60 and whose sum is –19. The necessary integers are –15 and –4. Write –19k as –15k – 4k and then factor by grouping.

$$= 6k^2 – 15k – 4k + 10$$

$$= (6k^2 – 15k) + (–4k + 10)$$

$$= 3k(2k – 5) –2(2k – 5)$$

$$= (2k – 5)(3k – 2)$$

CLASSROOM EXAMPLE 6 **Factoring Trinomials in $ax^2 + bx + c$ Form**

Factor each trinomial. Alternative Method

Solution:

$10x^2 + 17x + 3$

By trial and error, the following are factored.

$$= (5x + 1)(2x + 3)$$

$6r^2 + 13r – 5$

$$= (2r + 5)(3r – 1)$$

Use an alternative method for factoring trinomials.

Factoring $ax^2 + bx + c$ (GCF of a, b, c is 1)

***Step 1* Find pairs whose product is *a*.** Write all pairs of integer factors of a, the coefficient of the second-degree term.

***Step 2* Find pairs whose product is *c*.** Write all pairs of integer factors of c, the last term.

Step 3* Choose inner and outer terms.** Use FOIL and various combinations of the factors from ***Steps 1 and 2 until the necessary middle term is found.

If no such combinations exist, the trinomial is prime.

CLASSROOM EXAMPLE 7 **Factoring a Trinomial in Two Variables**

Factor $6m^2 + 7mn - 5n^2$.

Solution:

Try some possibilities.

$(6m + n)(m - 5n)$	$= 6m^2 - 29mn - 5n^2$	**No**
$(6m - 5n)(m + n)$	$= 6m^2 + mn - 5n^2$	**No**
$(3m + n)(2m - 5n)$	$= 6m^2 - 13mn - 5n^2$	**No**
$(3m + 5n)(2m - n)$	$= 6m^2 + 7mn - 5n^2$	**Yes**

The correct factoring is

$= (3m + 5n)(2m - n)$.

CLASSROOM EXAMPLE 8 **Factoring a Trinomial in $ax^2 + bx + c$ Form ($a < 0$)**

Factor $-2p^2 - 5p + 12$.

Solution:

First factor out -1, then proceed.

$= -1(2p^2 + 5p - 12)$

$= -1(p + 4)(2p - 3)$

$= -(p + 4)(2p - 3)$

CLASSROOM EXAMPLE 9 **Factoring a Trinomial with a Common Factor**

Factor $4m^3 + 2m^2 - 6m$.

Solution:

First, factor out the GCF, $2m$.

$= 2m(2m^2 + m - 3)$

Look for two integers whose product is $2(-3) = -6$ and whose sum is 1. The integers are 3 and -2.

$= 2m(2m^2 + 3m - 2m - 3)$

$= 2m[m(2m + 3) - 1(2m + 3)]$

$= 2m[(2m + 3)(m - 1)]$

$= 2m(2m + 3)(m - 1)$

CLASSROOM EXAMPLE 10 **Factoring a Polynomial by Substitution**

Factor $8(z + 5)^2 - 2(z + 5) - 3$.

Solution:

$= 8x^2 - 2x - 3$ **Let $x = z + 5$.**

$= (2x + 1)(4x - 3)$

Now replace x with $z + 5$.

$= [2(z + 5) + 1][4(z + 5) - 3]$

$= (2z + 10 + 1)(4z + 20 - 3)$

$= (2z + 11)(4z + 17)$

CLASSROOM EXAMPLE 11 **Factoring a Trinomial in $ax^4 + bx^2 + c$ Form**

Factor $6r^4 - 13r^2 + 5$.

Solution:

$= 6(r^2)^2 - 13r^2 + 5$

$= 6x^2 - 13x + 5$ **Let $x = r^2$.**

$= (3x - 5)(2x - 1)$ **Factor.**

$= (3r^2 - 5)(2r^2 - 1)$ ***$x = r^2$***