6.3 Special Factoring

Objectives

1 Factor a difference of squares.

2 Factor a perfect square trinomial.

3 Factor a difference of cubes.

4 Factor a sum of cubes.

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Factor a difference of squares.

Difference of Squares

$$x^2 - y^2 = (x + y)(x - y)$$

CLASSROOM EXAMPLE 1 **Factoring Differences of Squares**

Factor each polynomial.

$p^2 - 100$

Solution:

$= p^2 - 10^2$

$= (p + 10)(p - 10)$

$2x^2 - 18$

$= 2(x^2 - 9)$ **Factor out the GCF, 2.**

$= 2(x^2 - 3^2)$

$= 2(x + 3)(x - 3)$

$9a^2 - 16b^2$

$= (3a)^2 - (4b)^2$

$= (3a + 4b)(3a - 4b)$

CLASSROOM EXAMPLE 1 **Factoring Differences of Squares (cont'd)**

Factor each polynomial.

$(m + 3)^2 - 49z^2$

Solution:

$= (m + 3)^2 - (7z)^2$

$= (m + 3 + 7z)(m + 3 - 7z)$

$y^4 - 16$

$= (y^2)^2 - 4^2$

$= (y^2 + 4)(y^2 - 4)$

$= (y^2 + 4)(y + 2)(y - 2)$

Objective 2

Factor a prefect square trinomial.

Factor a perfect square trinomial.

Perfect Square Trinomial

$$x^2 + 2xy + y^2 = (x + y)^2$$

$$x^2 - 2xy + y^2 = (x - y)^2$$

CLASSROOM EXAMPLE 2 **Factoring Perfect Square Trinomials**

Factor the polynomial.

$49z^2 - 14z + 1$

Solution:

$= (7z)^2 - 14z + 1^2$

$= (7z - 1)^2$

Check. $2(7z)(-1) = -14z$, which is the middle term.

Thus, $49z^2 - 14z + 1 = (7z - 1)^2$.

CLASSROOM EXAMPLE 2 **Factoring Perfect Square Trinomials (cont'd)**

Factor the polynomial.

$9a^2 + 48ab + 64b^2$

Solution:

$= (3a)^2 + 48ab + (8b)^2$

$= (3a + 8b)^2$

Check. $2(3a)(8b) = 48ab$, which is the middle term.

Thus, $9a^2 + 48\text{ab} + 64\text{b}^2 = (3a)^2 + 48ab + (8b)^2 = (3a + 8b)^2$

CLASSROOM EXAMPLE 2 **Factoring Perfect Square Trinomials (cont'd)**

Factor the polynomial.

$x^2 - 2x + 1 - y^2$

Solution:

$= (x^2 - 2x + 1) - y^2$ **Factor by grouping.**

$= (x - 1)^2 - y^2$ **Factor the perfect square trinomial.**

This is the difference of two squares.

$= [(x - 1) + y][(x - 1) - y]$

$= (x - 1 + y)(x - 1 - y)$

Objective 3

Factor a difference of cubes.

Factor a difference of cubes.

Difference of Cubes

$$\boldsymbol{x^3 - y^3 = (x - y)(x^2 + xy + y^2)}$$

CLASSROOM EXAMPLE 3 **Factoring Differences of Cubes**

Factor the polynomial.

$x^3 - 1000$

Solution:

$A^3 - B^3 = (A - B)(A^2 + A \cdot B + B^2)$

$= x^3 - 10^3 = (x - 10)(x^2 + x \cdot 10 + 10^2)$

$= (x - 10)(x^2 + 10x + 100)$

CLASSROOM EXAMPLE 3 **Factoring Differences of Cubes (cont'd)**

Factor each polynomial.

$8k^3 - y^3$

Solution:

$= (2k)^3 - y^3$

$= (2k - y)[(2k)^2 + 2k(y) + y^2]$

$= (2k - y)(4k^2 + 2ky + y^2)$

$27m^3 - 64n^3$

$= (3m)^3 - (4n)^3$

$= (3m - 4n)[(3m)^2 + 3m(4n) + (4n)^2]$

$= (3m - 4n)(9m^2 + 12mn + 16n^2)$

Objective 4

Factor a sum of cubes.

Factor a sum of cubes.

Sum of Cubes

$$x^3 + y^3 = (x + y)(x^2 - xy + y^2)$$

The sign of the second term in the binomial factor of a sum or difference of cubes is ***always the same*** as the sign in the original polynomial. In the trinomial factor, the first and last terms are ***always positive.*** The sign of the middle term is ***the opposite*** *of* the sign of the second term in the binomial factor.

CLASSROOM EXAMPLE 4 **Factoring Sums of Cubes**

Factor each polynomial.

$8p^3 + 125$

Solution:

$= (2p)^3 + 5^3$

$= (2p + 5)[(2p)^2 - (2p)(5) + 5^2]$

$= (2p + 5)(4p^2 - 10p + 25)$

$64m^3 + 125n^3$

$= (4m)^3 + (5n)^3$

$= (4m + 5n)[(4m)^2 - 4m(5n) + (5n)^2]$

$= (4m + 5n)(16m^2 - 20mn + 25n^2)$

CLASSROOM EXAMPLE 4 **Factoring Sums of Cubes (cont'd)**

Factor each polynomial.

$2x^3 + 2000$

Solution:

$= 2(x^3 + 1000)$

$= 2(x^3 + 10^3)$

$= 2(x + 10)(x^2 - 10x + 10^2)$

$= 2(x + 10)(x^2 - 10x + 100)$

$(a - 4)^3 + b^3$

$= [(a - 4) + b][(a - 4)^2 - (a - 4)b + b^2$

$= (a - 4 + b)(a^2 - 8a + 16 - ab + 4b + b^2)$

Factor a sum of cubes.

Special Types of Factoring

Difference of Squares	$x^2 - y^2 = (x + y)(x - y)$
Perfect Square Trinomial	$x^2 + 2xy + y^2 = (x + y)^2$
	$x^2 - 2xy + y^2 = (x - y)^2$
Difference of Cubes	$x^3 - y^3 = (x - y)(x^2 + xy + y^2)$
Sum of Cubes	$x^3 + y^3 = (x + y)(x^2 - xy + y^2)$