6.4 A General Approach to Factoring

Objectives

1. Factor out any common factor.
2. Factor binomials.
3. Factor trinomials.
4. Factor polynomials of more than three terms.

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A General Approach to Factoring

Factoring a Polynomial

Step 1 **Factor out any common factor.**

Step 2 **If the polynomial is a binomial,** check to see if it is the difference of squares, a difference of cubes, or a sum of cubes.

If the polynomial is a trinomial, check to see if it is a perfect square trinomial. If it is not, factor as in **Section 6.2**.

If the polynomial has more than three terms, try to factor by grouping.

Step 3 ***Check the factored form by multiplying.***

CLASSROOM EXAMPLE 1 **Factoring Out a Common Factor**

Factor each polynomial.

$2x^3 + 10x^2 - 4x$

Solution:

The GCF is $2x$.

$= 2x(x^2 + 5x - 2)$

$12m(p - q) - 7n(p - q)$

The GCF is $(p - q)$

$= (p - q)(12m - 7n)$

Objective 2

Factor binomials.

Factor binomials.

Factoring a Binomial

For a **binomial** (two terms), check for the following:

Difference of Squares:

$$x^2 - y^2 = (x - y)(x + y)$$

Difference of Cubes:

$$x^3 - y^3 = (x - y)(x^2 + xy + y^2)$$

Sum of Cubes:

$$x^3 + y^3 = (x + y)(x^2 - xy + y^2)$$

CLASSROOM EXAMPLE 2 **Factoring Binomials**

Factor each binomial, if possible.

$36x^2 - y^2$

Solution:

Difference of squares

$= (6x)^2 - (y)^2$

$= (6x - y)(6x + y)$

$4t^2 + 1$

The binomial is prime. It is the sum of squares.

$125x^3 - 27y^3 =$

$= (5x - 3y)[(5x)^2 + (5x)(3y) + (3y)^2]$

$= (5x - 3y)(25x^2 + 15xy + 9y^2)$

Factor trinomials.

Factoring a Trinomial

For a **trinomial** (three terms), decide whether it is a perfect square trinomial of either of these forms

$$x^2 + 2xy + y^2 = (x + y)^2$$

or

$$x^2 - 2xy + y^2 = (x - y)^2$$

If not, use the methods of **Section 6.2.**

CLASSROOM EXAMPLE 3 **Factoring Trinomials**

Factor each trinomial.

$16m^2 + 56m + 49$

Solution:

$= (4m + 7)^2$ **Perfect square trinomial**

$8t^2 - 13t + 5$

Two integer factors whose sum is 8(5) = 40 and whose sum is –13 are –5 and –8.

$= 8t^2 - 5t - 8t + 5$
$= t(8t - 5) - 1(8t - 5)$
$= (8t - 5)(t - 1)$

CLASSROOM EXAMPLE 3 **Factoring Trinomials (cont'd)**

Factor the trinomial.

$6x^2 - 3x - 63$

Solution:

Factor out the GCF of 3.

$= 3(2x^2 - x - 21)$

Two factors whose product is 2(–21) = – 42 and whose sum is –1 are –7 and 6.

$= 3[2x^2 - 7x + 6x - 21]$
$= 3[x(2x - 7) + 3(2x - 7)]$
$= 3(2x - 7)(x + 3)$

Objective 3

Factor polynomials of more than three terms.

CLASSROOM EXAMPLE 4 **Factoring Polynomials with More than Three Terms**

Factor each polynomial.

$p^3 - 2pq^2 + p^2q - 2q^3$

Solution:

$= (p^3 - 2pq^2) + (p^2q - 2q^3)$
$= p(p^2 - 2q^2) + q(p^2 - 2q^2)$
$= (p^2 - 2q^2)(p + q)$

$9x^2 + 24x + 16 - y^2$

$= (9x^2 + 24x + 16) - y^2$
$= (3x + 4)^2 - y^2$
$= [(3x + 4) + y)][(3x + 4) - y)]$
$= (3x + 4 + y)(3x + 4 - y)$

CLASSROOM EXAMPLE 4 **Factoring Polynomials with More than Three Terms (cont'd)**

Factor the polynomial.

$64a^3 + 16a^2 + b^3 - b^2$

Solution:

$= (64a^3 + b^3) + (16a^2 - b^2)$
$= [(4a)^3 + b^3] + [(4a)^2 - b^2]$
$= \{(4a + b)[(4a)^2 - 4ab + b^2]\} + [(4a + b)(4a - b)]$
$= [(4a + b)(16a^2 - 4ab + b^2)] + [(4a + b)(4a - b)]$
$= (4a + b)(16a^2 - 4ab + b^2 + 4a - b)$