6.5 Solving Equations by Factoring

Objectives

1 Learn and use the zero-factor property.

2 Solve applied problems that require the zero-factor property.

3 Solve a formula for a specified variable, where factoring is necessary.

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CLASSROOM EXAMPLE 1 **Using the Zero-Factor Property to Solve an Equation**

Solve $(8x + 3)(2x + 1) = 0$.

Solution:

By the zero-factor property, either

$8x + 3 = 0$ or $2x + 1 = 0$

$$8x+3=0 \quad \text{or} \quad 2x+1=0$$

$$8x=-3 \quad \text{or} \quad 2x=-1$$

$$x=-\frac{3}{8} \quad \text{or} \quad x=-\frac{1}{2}$$

Check the two solutions by substitution into the ***original*** equation.

Learn and use the zero-factor property.

Zero-Factor Property

If two numbers have a product of 0, then at least one of the numbers must be 0.

That is, if $ab = 0$, then either $a = 0$ or $b = 0$.

CAUTION ***The zero-factor property works only for a product equal to 0.***

CLASSROOM EXAMPLE 1 **Using the Zero-Factor Property to Solve an Equation (cont'd)**

Check the solutions.

$(8x + 3)(2x + 1) = 0$.

$$\left((8)\left(-\frac{3}{8}\right)+3\right)\left((2)\left(-\frac{3}{8}\right)+1\right)=0$$

$$(-3+3)\left(-\frac{6}{8}+\frac{8}{8}\right)=0$$

$$0=0$$

True

$$\left((8)\left(-\frac{1}{2}\right)+3\right)\left((2)\left(-\frac{1}{2}\right)+1\right)=0$$

$$(-4+3)(0)=0$$

$$0=0$$

True

Both solutions check; the solution set is $\left\{-\frac{3}{8},-\frac{1}{2}\right\}$.

Learn and use the zero-factor property.

Quadratic Equation

An equation that can be written in the form

$$ax^2 + bx + c = 0$$

where a, b, and c are real numbers, with $a \neq 0$, is a **quadratic equation.** This form is called **standard form**.

Learn and use the zero-factor property.

Solving a Quadratic Equation by Factoring

Step 1 **Write in standard form.** Rewrite the equation if necessary so that one side is 0.

Step 2 **Factor** the polynomial.

Step 3 **Use the zero-factor property.** Set each variable factor equal to 0.

Step 4 **Find the solution(s).** Solve each equation formed in ***Step 3.***

Step 5 **Check** each solution in the ***original*** equation.

CLASSROOM EXAMPLE 2 — Solving Quadratic Equations by Factoring

Solve the equation.

$3x^2 - x = 4$

Solution:

Step 1 **Standard form.** $3x^2 - x - 4 = 0$

Step 2 **Factor.** $(3x - 4)(x + 1) = 0$

Step 3 **Zero-factor.** $3x - 4 = 0$ or $x + 1 = 0$

Step 4 **Solve.** $3x = 4$, $x = \frac{4}{3}$ | $x = -1$

Step 5 **Check.**

$3x^2 - x = 4$; $3\left(\frac{4}{3}\right)^2 - \frac{4}{3} = 4$; $4 = 4$ **True**

$3x^2 - x = 4$; $3(-1)^2 + 1 = 4$; $4 = 4$ **True**

The solution set is $\left\{-1, \frac{4}{3}\right\}$.

CLASSROOM EXAMPLE 2 — Solving Quadratic Equations by Factoring (cont'd)

Solve the equation.

$16m^2 + 24m + 9 = 0$

Solution:

Step 1 **Standard form.** Already in standard form.

Step 2 **Factor.** $(4m + 3)^2 = 0$

Step 3 **Zero-factor.** $4m + 3 = 0$

Step 4 **Solve.** $4m = -3$, $m = \frac{-3}{4}$

Step 5 **Check.** $16\left(\frac{-3}{4}\right)^2 + 24\left(\frac{-3}{4}\right) + 9 = 0$

$9 - 18 + 9 = 0$

$0 = 0$ **True**

The solution set is $\left\{-\frac{3}{4}\right\}$.

CLASSROOM EXAMPLE 3 — Solving a Quadratic Equation with a Missing Constant Term

Solve $x^2 + 12x = 0$

Solution:

Factor. $x(x + 12) = 0$

$x = 0$ or $x + 12 = 0$

$x = 0$ or $x = -12$

Check.

$x^2 + 12x = 0$; $0^2 + 12(0) = 0$; **True** $0 = 0$

$x^2 + 12x = 0$; $(-12)^2 + 12(-12) = 0$; $0 = 0$ **True**

The solution set is $\{-12, 0\}$.

CLASSROOM EXAMPLE 4 — Solving a Quadratic Equation with a Missing Linear Term

Solve $5x^2 - 80 = 0$

Solution:

$5x^2 - 80 = 5(x^2 - 16)$ **Factor out 5.**

$= 5(x - 4)(x + 4)$ **Factor.**

$x - 4 = 0$ or $x + 4 = 0$ **Solve.**

$x = 4$ or $x = -4$

Check that the solution set is $\{-4, 4\}$.

CLASSROOM EXAMPLE 5 — Solving an Equation That Requires Rewriting

Solve $(x + 6)(x - 2) = 2 + x - 10$.

Solution:

$x^2 + 4x - 12 = x - 8$ **Multiply.**

$x^2 + 3x - 4 = 0$ **Standard form.**

$(x - 1)(x + 4) = 0$ **Factor.**

$x - 1 = 0$ or $x + 4 = 0$

$x = 1$ or $x = -4$

Check that the solution set is $\{-4, 1\}$.

CLASSROOM EXAMPLE 6 — Solving an Equation of Degree 3

Solve $3x^3 + x^2 = 4x$

Solution:

$3x^3 + x^2 - 4x = 0$ **Standard form.**

$x(3x^2 + x - 4) = 0$ **Factor out *n*.**

$x(3x + 4)(x - 1) = 0$

$x = 0$ or $3x + 4 = 0$ or $x - 1 = 0$

$x = -\frac{4}{3}$ $x = 1$

Check that the solution set is $\left\{-\frac{4}{3}, 0, 1\right\}$.

CLASSROOM EXAMPLE 7

Using a Quadratic Equation in an Application

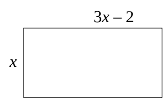
The length of a room is 2 m less than three times the width. The area of the room is 96 m². Find the width of the room.

Solution:

Step 1 **Read** the problem again. There will be one answer.

Step 2 **Assign a variable.**

Let x = the width of the room.
Let $3x - 2$ = the length of the room.
The area is 96.



CLASSROOM EXAMPLE 7

Using a Quadratic Equation in an Application (cont'd)

Step 3 **Write an equation.** The area of a rectangle is the length times the width.

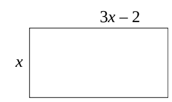
$A = lw$
$96 = (3x - 2)x$

Step 4 **Solve.**

$$96 = (3x - 2)x$$
$$96 = 3x^2 - 2x$$
$$0 = 3x^2 - 2x - 96$$
$$0 = (3x + 16)(x - 6)$$

$3x + 16 = 0$ or $x - 6 = 0$

$x = -\frac{16}{3}$ $\quad x = 6$



CLASSROOM EXAMPLE 7

Using a Quadratic Equation in an Application (cont'd)

Step 5 **State the answer.**

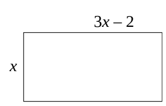
A distance cannot be negative, so reject $-\frac{16}{3}$ *as a solution.*

The only possible solution is 6, so the width of the room is 6 m.

Step 6 **Check.**

The length is 2 m less than three times the width, the length would be $3(6) - 2 = 16$.

The area is $6 \cdot 16 = 96$, as required.



CAUTION ***When application lead to quadratic equations, a solution of the equation may not satisfy the physical requirements of the problem. Reject such solutions as valid answers.***

CLASSROOM EXAMPLE 8

Using a Quadratic Function in an Application

If a small rocket is launched vertically upward from ground level with an initial velocity of 128 ft per sec, then its height in feet after t seconds is a function defined by $h(t) = -16t^2 + 128t$ if air resistance is neglected. How long will it take the rocket to reach a height of 256 feet?

Solution:

We let $h(t) = 256$ and solve for t.

$$256 = -16t^2 + 128t$$
$$16t^2 - 128t + 256 = 0 \qquad \textbf{Divide by 16.}$$
$$t^2 - 8t + 16 = 0$$
$$(t - 4)^2 = 0$$
$$t - 4 = 0$$
$$t = 4$$

It will reach a height of 256 feet in 4 seconds.

Objective 3

Solve a formula for a specified variable, where factoring is necessary.

CLASSROOM EXAMPLE 9

Using Factoring to Solve for a Specified Variable

Solve the formula for W.

$A = 2HW + 2LW + 2LH$

Solution:

$$A - 2LH = 2HW + 2LW$$
$$A - 2LH = W(2H + 2L)$$
$$\frac{A - 2LH}{2H + 2L} = W \quad \text{or} \quad W = \frac{A - 2LH}{2H + 2L}$$

CAUTION ***We must write the expression so that the specified variable is a factor. Then we can divide by its coefficient in the final step.***