

7.2 Adding and Subtracting Rational Expressions

Objectives

- 1 Add and subtract rational expressions with the same denominator.
- 2 Find a least common denominator.
- 3 Add and subtract rational expressions with different denominators.

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Objective 1

Add and subtract rational expressions with the same denominator.

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Add and subtract rational expressions with the same denominator.

Adding or Subtracting Rational Expressions

Step 1 If the denominators are the same, add or subtract the numerators. Place the result over the common denominator.

If the denominators are different, first find the least common denominator. Write all rational expressions with this least common denominator, and then add or subtract the numerators. Place the result over the common denominator.

Step 2 Simplify. Write all answers in lowest terms.

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CLASSROOM EXAMPLE 1

Adding and Subtracting Rational Expressions Same Denominator

Add or subtract as indicated.

Solution:

$$\frac{7x}{9} + \frac{y}{9} = \frac{7x+y}{9}$$

$$\frac{8}{3x^3} - \frac{14}{3x^3} = \frac{8-14}{3x^3} = \frac{-6}{3x^3} = \frac{-2}{x^3}$$

$$\frac{r}{r^2-t^2} + \frac{t}{r^2-t^2} = \frac{r+t}{r^2-t^2} = \frac{r+t}{(r+t)(r-t)} = \frac{1}{r-t}$$

$$\begin{aligned} \frac{6}{x^2+3x-18} + \frac{x}{x^2+3x-18} &= \frac{6+x}{x^2+3x-18} \\ &= \frac{6+x}{(x+6)(x-3)} = \frac{1}{x-3} \end{aligned}$$

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Objective 2

Find a least common denominator.

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Find a least common denominator.

Finding the Least Common Denominator

Step 1 Factor each denominator.

Step 2 Find the least common denominator. The LCD is the product of all of the different factors from each denominator, with each factor raised to the **greatest** power that occurs in any denominator.

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CLASSROOM EXAMPLE 2 Finding Least Common Denominators

Find the LCD for each group of denominators.

$10a^3b^5, 15a^2b^6$

Solution:

Factor. $10a^3b^5 = 2 \cdot 5 \cdot a^3 \cdot b^5$
 $15a^2b^6 = 3 \cdot 5 \cdot a^2 \cdot b^6$

LCD $= 2 \cdot 3 \cdot 5 \cdot a^3 \cdot b^6 = 30a^3b^6$

$z, z + 6$

Each denominator is already factored.

LCD $= z(z + 6)$

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CLASSROOM EXAMPLE 2 Finding Least Common Denominators (cont'd)

Find the LCD for each group of denominators.

$m^2 - 16, m^2 + 8m + 16$

Solution:

Factor. $m^2 - 16 = (m + 4)(m - 4)$
 $m^2 + 8m + 16 = (m + 4)^2$

LCD $= (m + 4)^2(m - 4)$

$x^2 - 2x + 1, x^2 - 4x + 3, 4x - 4$

$x^2 - 2x + 1 = (x - 1)(x - 1)$
 $x^2 - 4x + 3 = (x - 1)(x - 3)$
 $4x - 4 = 4(x - 1)$

LCD $= 4(x - 1)^2(x - 3)$

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Objective 3

Add and subtract rational expressions with different denominators.

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CLASSROOM EXAMPLE 3 Adding and Subtracting Rational Expressions (Different Denominators)

Add or subtract as indicated.

Solution:

$$\frac{6}{m} + \frac{1}{4m} = \frac{6 \cdot 4}{m \cdot 4} + \frac{1}{4m} = \frac{24}{4m} + \frac{1}{4m} = \frac{24 + 1}{4m} = \frac{25}{4m}$$

$$\frac{2}{y} - \frac{1}{y + 4} = \frac{2(y + 4)}{y(y + 4)} - \frac{(1)y}{(y + 4)y} = \frac{2(y + 4) - y}{y(y + 4)}$$

$$= \frac{2y + 8 - y}{y(y + 4)} = \frac{y + 8}{y(y + 4)}$$

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CLASSROOM EXAMPLE 4 Subtracting Rational Expressions

Subtract.

$$\frac{5x + 7}{2x + 7} - \frac{-x - 14}{2x + 7}$$

Solution:

The denominators are already the same for both rational expressions. The subtraction sign must be applied to both terms in the numerator of the second rational expression.

$$= \frac{5x + 7 - (-x - 14)}{2x + 7} = \frac{5x + 7 + x + 14}{2x + 7} = \frac{6x + 21}{2x + 7}$$

$$= \frac{3(2x + 7)}{2x + 7} = 3$$

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CLASSROOM EXAMPLE 4 Subtracting Rational Expressions (cont'd)

Subtract.

$$\frac{2}{r - 2} - \frac{r + 3}{r - 1}$$

Solution:

The LCD is $(r - 2)(r - 1)$.

$$= \frac{2(r - 1)}{(r - 2)(r - 1)} - \frac{(r + 3)(r - 2)}{(r - 1)(r - 2)} = \frac{2r - 2 - (r^2 + r - 6)}{(r - 2)(r - 1)}$$

$$= \frac{2r - 2 - r^2 - r + 6}{(r - 2)(r - 1)} = \frac{-r^2 + r + 4}{(r - 2)(r - 1)}$$

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**CLASSROOM
EXAMPLE 5**

Adding and Subtracting Rational Expressions (Denominators Are Opposites)

Add.

$$\frac{2}{x-3} + \frac{1}{3-x}$$

Solution:

To get a common denominator of $x-3$, multiply both the numerator and denominator of the second expression by -1 .

$$= \frac{2}{x-3} + \frac{1(-1)}{(3-x)(-1)} = \frac{2}{x-3} + \frac{-1}{x-3} = \frac{2+(-1)}{x-3} = \frac{1}{x-3}$$

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**CLASSROOM
EXAMPLE 6**

Adding and Subtracting Three Rational Expressions

Add and subtract as indicated.

$$\frac{4}{x-5} + \frac{-2}{x} - \frac{10}{x^2-5x}$$

Solution:

$$\begin{aligned} &= \frac{4}{x-5} + \frac{-2}{x} - \frac{10}{x(x-5)} \\ &= \frac{4x}{(x-5)x} + \frac{-2(x-5)}{x(x-5)} - \frac{10}{x(x-5)} \\ &= \frac{4x + (-2)(x-5) - 10}{x(x-5)} \\ &= \frac{4x - 2x + 10 - 10}{x(x-5)} = \frac{2x}{x(x-5)} = \frac{2}{x-5} \end{aligned}$$

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**CLASSROOM
EXAMPLE 7**

Subtracting Rational Expressions

Subtract.

$$\frac{-a}{a^2+3a-4} - \frac{4a}{a^2+7a+12}$$

Solution:

$$= \frac{-a}{(a+4)(a-1)} - \frac{4a}{(a+4)(a+3)}$$

LCD is $(a+4)(a-1)(a+3)$.

$$= \frac{-a(a+3)}{(a+4)(a-1)(a+3)} - \frac{4a(a-1)}{(a+4)(a-1)(a+3)}$$

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**CLASSROOM
EXAMPLE 7**

Subtracting Rational Expressions (cont'd)

$$= \frac{-a(a+3) - 4a(a-1)}{(a+4)(a-1)(a+3)}$$

$$= \frac{-a^2 - 3a - 4a^2 + 4a}{(a+4)(a-1)(a+3)}$$

Distributive property

$$= \frac{-5a^2 + a}{(a+4)(a-1)(a+3)}$$

Combine terms in the numerator.

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**CLASSROOM
EXAMPLE 8**

Adding Rational Expressions

Add.

$$\frac{4}{p^2-6p+9} + \frac{1}{p^2+2p-15}$$

Solution:

$$= \frac{4}{(p-3)(p-3)} + \frac{1}{(p+5)(p-3)}$$

LCD is $(p-3)^2(p+5)$.

$$= \frac{4(p+5)}{(p-3)^2(p+5)} + \frac{1(p-3)}{(p+5)(p-3)^2}$$

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**CLASSROOM
EXAMPLE 8**

Adding Rational Expressions (cont'd)

$$= \frac{4(p+5) + 1(p-3)}{(p-3)^2(p+5)}$$

$$= \frac{4p + 20 + p - 3}{(p-3)^2(p+5)}$$

Distributive property

$$= \frac{5p + 17}{(p-3)^2(p+5)}$$

Combine terms in the numerator.

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