7.5 Applications of Rational Expressions

Objectives

1. Find the value of an unknown variable in a formula.
2. Solve a formula for a specified variable.
3. Solve applications by using proportions.
4. Solve applications about distance, rate, and time.
5. Solve applications about work rates.

Objective 1

Find the value of an unknown variable in a formula.

CLASSROOM EXAMPLE 1 Finding the Value of a Variable in a Formula

Use the formula $\frac{1}{f}=\frac{1}{p}+\frac{1}{q}$ to find *p* if *f* = 15 cm and *q* = 25 cm.

Solution:

$$\frac{1}{f}=\frac{1}{p}+\frac{1}{q}$$

$$\frac{1}{15}=\frac{1}{p}+\frac{1}{25}$$ **Let *f* = 15 and *q* = 25.**

$$75p\cdot\frac{1}{15}=75p\left(\frac{1}{p}+\frac{1}{25}\right)$$ **Multiply by the LCD, 75*p*.**

$$5p=75+3p$$

$$2p=75$$

$$p=\frac{75}{2}$$

Objective 2

Solve a formula for a specified variable.

CLASSROOM EXAMPLE 2 Finding a Formula for a Specified Variable

Solve $\frac{3}{p}+\frac{3}{q}=\frac{5}{r}$ for *q*.

Solution:

$$pqr\left(\frac{3}{p}+\frac{3}{q}\right)=pqr\cdot\frac{5}{r}$$ **Multiply by the LCD, *pqr*.**

$$3qr+3pr=5pq$$ **Distributive property.**

$$3pr=5pq-3qr$$ **Subtract 3*qr* to get all *q* terms on same side of equation.**

$$3pr=q(5p-3r)$$ **Factor out *q*.**

$$\frac{3pr}{5p-3r}=q \quad \text{or} \quad q=\frac{3pr}{5p-3r}$$ **Divide.**

CLASSROOM EXAMPLE 3 Solving a Formula for a Specified Variable

Solve $A=\frac{Rr}{R+r}$ for *R*.

Solution:

$$(R+r)A=(R+r)\left(\frac{Rr}{R+r}\right)$$

$$A(R+r)=Rr$$

$$AR+Ar=Rr$$

$$AR-Rr=-Ar$$

$$R(A-r)=-Ar$$

$$R=\frac{-Ar}{A-r} \quad \text{or} \quad \frac{Ar}{r-A}$$

Solve applications by using proportions.

A **ratio** is a comparison of two quantities. The ratio of *a* to *b* may be written in any of the following ways:

a to b, $\quad a{:}b, \quad$ or $\quad \frac{a}{b}.$

Ratio of *a* to *b*

Ratios are usually written as quotients in algebra. A **proportion** is a statement that two ratios are equal, such as

$$\frac{a}{b} = \frac{c}{d}.$$

Proportion

CLASSROOM EXAMPLE 4 **Solving a Proportion**

In 2008, approximately 9.9% (that is, 9.9 of every 100) of the 74,510,000 children under 18 yr of age in the United States had no health insurance. How many such children were uninsured? (*Source:* U.S. Census Bureau.)

Solution:

Step 1 **Read** the problem.

Step 2 **Assign a variable.**
Let x = the number (in millions) who had no health insurance.

Step 3 **Write an equation.** To get an equation, set up a proportion.

$$\frac{9.9}{100} = \frac{x}{74{,}510{,}000}$$

CLASSROOM EXAMPLE 4 **Solving a Proportion (cont'd)**

Step 4 **Solve.**

$$\frac{9.9}{100} = \frac{x}{74{,}510{,}000}$$

$$74{,}510{,}000 \cdot \frac{9.9}{100} = 74{,}510{,}000 \cdot \frac{x}{74{,}510{,}000}$$

$$745{,}100 \cdot 9.9 = x$$

$$x = 7{,}376{,}490$$

Step 5 **State the answer**. There were 7,376,490 children under 18 years of age in the United States with no health insurance in 2008.

Step 6 **Check**. The ratio $\frac{7{,}376{,}490}{74{,}510{,}000} = \frac{9.9}{100}.$

CLASSROOM EXAMPLE 5 **Solving a Proportion Involving Rates**

Lauren's car uses 15 gal of gasoline to drive 390 mi. She has 6 gal of gasoline in the car, and she wants to know how much more gasoline she will need to drive 800 mi. If we assume that the car continues to use gasoline at the same rate, how many more gallons will she need?

Solution:

Step 1 **Read** the problem.

Step 2 **Assign a variable.**
Let x = the additional number of gallons needed.

CLASSROOM EXAMPLE 5 **Solving a Proportion Involving Rates (cont'd)**

Step 3 **Write an equation.** She knows that she can drive 390 miles with 15 gallons of gasoline. She wants to drive 800 miles using (6 + x) gallons of gasoline. Set up a proportion.

Step 4 **Solve.** $\quad \frac{390}{15} = \frac{800}{6+x}$

$$\frac{26}{1} = \frac{800}{6+x} \quad \textbf{Reduce.}$$

$$26(6+x) = 800$$

$$156 + 26x = 800$$

$$26x = 644$$

$$x = \frac{644}{26}$$

$$x \approx 24.8$$

CLASSROOM EXAMPLE 5 **Solving a Proportion Involving Rates (cont'd)**

Step 5 **State the answer**. She will need about 24.8 more gallons of gasoline.

Step 6 **Check**. The 6 gallons + 24.8 gallons equals 30.8 gallons. Check the rates (miles/gallon).

$$\frac{390}{15} = 26 \text{ mpg} \qquad \frac{800}{30.8} \approx 26 \text{ mpg}$$

The rates are approximately equal, so the solution is correct.

CLASSROOM EXAMPLE 6 Solving a Problem about Distance, Rate, and Time

A plane travels 100 mi against the wind in the same time that it takes to travel 120 mi with the wind. The wind speed is 20 mph. Find the speed of the plane in still air.

Solution:

Step 1 **Read** the problem.

We must find the speed of the plane in still air.

Step 2 **Assign a variable.**

Let x = the speed of the plane in still air.
Use $d = rt$, to complete the table (next slide).

CLASSROOM EXAMPLE 6 Solving a Problem about Distance, Rate, and Time (cont'd)

	d	*r*	*t*
Against Wind	100	$x - 20$	$\frac{100}{x-20}$
With Wind	120	$x + 20$	$\frac{100}{x+20}$

Step 3 **Write an equation.** Since the time against the wind equals the time with the wind, we set up this equation.

$$\frac{100}{x-20} = \frac{120}{x+20}$$

CLASSROOM EXAMPLE 6 Solving a Problem about Distance, Rate, and Time (cont'd)

Step 4 **Solve.**

Multiply by the LCD $(x - 20)(x + 20)$.

$$\frac{100}{x-20} = \frac{120}{x+20}$$

$$(x-20)(x+20)\frac{100}{x-20} = (x-20)(x+20)\frac{120}{x+20}$$

$$100(x+20) = 120(x-20)$$

$$100x + 2000 = 120x - 2400$$

$$4400 = 20x$$

$$220 = x$$

CLASSROOM EXAMPLE 6 Solving a Problem about Distance, Rate, and Time (cont'd)

Step 5 **State the answer.**

The speed of the airplane is 220 mph in still air.

Step 6 **Check.**

$$\frac{100}{220-20} = \frac{120}{220+20}$$

$$\frac{100}{200} = \frac{120}{240}$$

$$\frac{1}{2} = \frac{1}{2}$$

CLASSROOM EXAMPLE 7 Solving a Problem about Distance, Rate, and Time

Dona Kenly drove 300 mi north from San Antonio, mostly on the freeway. She usually averaged 55 mph, but an accident slowed her speed through Dallas to 15 mph. If her trip took 6 hr, how many miles did she drive at the reduced rate?

Solution:

Step 1 **Read** the problem.

We must find how many miles she drove at the reduced speed.

Step 2 **Assign a variable.**

Let x = the distance at reduced speed.
Use $d = rt$, to complete the table (next slide).

CLASSROOM EXAMPLE 7 Solving a Problem about Distance, Rate, and Time (cont'd)

	d	*r*	*t*
Normal Speed	$300 - x$	55	$\frac{300-x}{55}$
Reduced Speed	x	15	$\frac{x}{15}$

Step 3 **Write an equation.**

Time on freeway	plus	Time at reduced speed	equals 6 hr.
$\frac{300-x}{55}$	$+$	$\frac{x}{15}$	$= 6$

CLASSROOM EXAMPLE 7 Solving a Problem about Distance, Rate, and Time (cont'd)

Step 4 **Solve.**

Multiply by the LCD, 165.

$$165\left(\frac{300-x}{55}+\frac{x}{15}\right)=165\cdot 6$$
$$3(300-x)+11x=990$$
$$900-3x+11x=990$$
$$8x=90$$
$$x=\frac{90}{8}\ \text{or}\ 11\tfrac{1}{4}$$

Step 5 **State the answer.** She drove 11 ¼ miles at reduced speed.

Step 6 **Check.** The check is left to the student.

Solve applications about work rates.

Objective 5

PROBLEM-SOLVING HINT

People work at different rates. If the letters *r, t,* and *A* represent the rate at which work is done, the time required, and the amount of work accomplished, respectively, then *A* = *rt*. Notice the similarity to the distance formula, *d* = *rt.*

Amount of work can be measured in terms of jobs accomplished. Thus, if 1 job is completed, then *A* = 1, and the formula gives the rate as

$$1=rt,\quad\text{or}\quad r=\frac{1}{t}.$$

Solve applications about work rates.

Rate of Work

If a job can be accomplished in *t* units of time, then the rate of work is

$$\frac{1}{t}\ \text{job per unit of time.}$$

CLASSROOM EXAMPLE 8 Solving a Problem about Work

Stan needs 45 minutes to do the dishes, while Bobbie can do them in 30 minutes. How long will it take them if they work together?

Solution:

Step 1 **Read** the problem.

We must determine how long it will take them working together to wash the dishes.

Step 2 **Assign a variable.**

Let *x* = the time it will take them working together.

	Rate	Time Working Together	Fractional Part of the Job Done
Stan	$\frac{1}{45}$	*x*	$\frac{1}{45}x$
Bobbie	$\frac{1}{30}$	*x*	$\frac{1}{30}x$

CLASSROOM EXAMPLE 8 Solving a Problem about Work (cont'd)

Step 3 **Write an equation.**

Part done by Stan	plus	Part done by Bobbie	equals	1 whole job.
$\frac{1}{45}x$	$+$	$\frac{1}{30}x$	$=$	1

Step 4 **Solve.** Multiply by the LCD, 90.

$$90\left(\frac{1}{45}x+\frac{1}{30}x\right)=90\cdot 1$$
$$2x+3x=90$$
$$5x=90$$
$$x=18$$

CLASSROOM EXAMPLE 8 Solving a Problem about Work (cont'd)

Step 5 **State the answer.**

It will take them 18 minutes working together.

Step 6 **Check.**

The check is left to the student.