7.1 Rational Expressions and Functions; Multiplying and Dividing

Objectives

1. Define rational expressions.
2. Define rational functions and describe their domains.
3. Write rational expressions in lowest terms.
4. Multiply rational expressions.
5. Find reciprocals of rational expressions.
6. Divide rational expressions.

Define rational expressions.

A **rational expression** or ***algebraic fraction,*** is the quotient of two polynomials, again with the denominator not 0.

For example:

$$\frac{x}{y},\quad \frac{-a}{4},\quad \frac{m+4}{m+2},\quad \frac{8x^2-2x+5}{4x^2+5x},\quad x^5\left(\text{or } \frac{x^5}{1}\right)$$

are all rational expressions. Rational expressions are elements of the set

$$\left\{\frac{P}{Q}\,\middle|\, P \text{ and } Q \text{ are polynomials, with } Q \neq 0\right\}.$$

Define rational functions and describe their domains.

A function that is defined by a quotient of polynomials is called a **rational function** and has the form

$$f(x) = \frac{P(x)}{Q(x)},\ \text{where } Q(x) \neq 0.$$

The domain of the rational function consists of all real numbers except those that make *Q*(*x*)—that is, the denominator—equal to 0.

For example, the domain of $f(x) = \frac{2}{\underbrace{x-5}_{\text{Cannot equal 0}}}$

includes all real numbers except 5, because 5 would make the denominator equal to 0.

CLASSROOM EXAMPLE 1 Finding Domains of Rational Functions

For each rational function, find all numbers that are not in the domain. Then give the domain in set-builder notation.

$$f(x) = \frac{x+6}{x^2-x-6} \qquad f(x) = \frac{3+2x}{5}$$

Solution:

$$x^2 - x - 6 = 0$$
$$(x + 2)(x - 3) = 0$$
$$x + 2 = 0 \text{ or } x - 3 = 0$$
$$x = -2 \text{ or } \quad x = 3$$
$$\{x \mid x \neq -2, 3\}$$

The denominator 5 cannot ever be 0, so the domain includes all real numbers. $(-\infty, \infty)$

Write rational expressions in lowest terms.

Fundamental Property of Rational Numbers

If $\frac{a}{b}$ is a rational number and if *c* is any nonzero real number, then

$$\frac{a}{b} = \frac{ac}{bc}.$$

That is, the numerator and denominator of a rational number may either be multiplied or divided by the same ***nonzero number*** without changing the value of the rational number.

A rational expression is a quotient of two polynomials. Since the value of a polynomial is a real number for every value of the variable for which it is defined, any statement that applies to rational numbers will also apply to rational expressions.

Write rational expressions in lowest terms.

Writing a Rational Expression in Lowest Terms

Step 1 **Factor** both the numerator and denominator to find their greatest common factor (GCF).

Step 2 **Apply the fundamental property.** Divide out common factors.

CAUTION ***Be careful! When using the fundamental property of rational numbers, only common factors may be divided. Remember to factor before writing a fraction in lowest terms.***

CLASSROOM EXAMPLE 2 — **Writing Rational Expressions in Lowest Terms**

Write each rational expression in lowest terms.

$$\frac{y^2+2y-3}{y^2-3y+2} \qquad \frac{y+2}{y^2+4} \qquad \frac{1+p^3}{1+p}$$

Solution:

$$=\frac{(y+3)(y-1)}{(y-2)(y-1)}$$

$$=\frac{(y+3)}{(y-2)}\cdot 1$$

$$=\frac{(y+3)}{(y-2)}$$

The denominator cannot be factored, so this expression cannot be simplified further and is in lowest terms.

$$=\frac{(1+p)(1-p+p^2)}{(1+p)}$$

$$=1-p+p^2$$

CLASSROOM EXAMPLE 3 — **Writing Rational Expressions in Lowest Terms**

Write each rational expression in lowest terms.

$$\frac{r-1}{1-r} \qquad \frac{p-2}{4-p^2}$$

Solution:

$$\frac{r-1}{1-r}=\frac{r-1}{-1(r-1)}$$

$$=\frac{1}{-1}=-1$$

$$=\frac{p-2}{-1(p^2-4)}$$

$$=\frac{p-2}{-1(p-2)(p+2)}$$

$$=\frac{1}{-1(p+2)} \qquad =\frac{-1}{p+2}$$

Write rational expressions in lowest terms.

Quotient of Opposites

In general, if the numerator and the denominator of a rational expression are opposites, then the expression equals –1.

$$\frac{q-7}{7-q}=-1 \quad \text{and} \quad \frac{-5a+2b}{5a-2b}=-1$$

Numerator and denominator in each expression are opposites.

$$\frac{r-2}{r+2}$$

Numerator and denominator are ***not*** opposites.

Multiply rational expressions.

Multiplying Rational Expressions

Step 1 **Factor** all numerators and denominators as completely as possible.

Step 2 **Apply the fundamental property.**

Step 3 **Multiply** the numerators and multiply the denominators.

Step 4 **Check** to be sure that the product is in lowest terms.

CLASSROOM EXAMPLE 4 — **Multiplying Rational Expressions**

Multiply.

$$\frac{c^2+2c}{c^2-4}\cdot\frac{c^2-4c+4}{c^2-c} \qquad \frac{m^2-16}{m+2}\cdot\frac{1}{m+4}$$

Solution:

$$=\frac{c(c+2)}{(c-2)(c+2)}\cdot\frac{(c-2)(c-2)}{c(c-1)}$$

$$=\frac{(c-2)}{(c-1)}$$

$$=\frac{(m-4)(m+4)}{m+2}\cdot\frac{1}{m+4}$$

$$=\frac{m-4}{m+2}$$

Objective 5

Find reciprocals of rational expressions.

Find reciprocals of rational expressions.

Finding the Reciprocal

To find the reciprocal of a nonzero rational expression, invert the rational expression.

Two rational expressions are reciprocals of each other if they have a product of 1. Recall that 0 has no reciprocal.

Divide rational expressions.

Dividing Rational Expressions

To divide two rational expressions, ***multiply*** the first (the ***dividend***) by the reciprocal of the second (the ***divisor***).

CLASSROOM EXAMPLE 5 **Dividing Rational Expressions**

Divide.

$$\frac{5p+2}{6} \div \frac{15p+6}{5}$$

$$\frac{q^2+2q}{5+q} \div \frac{4-q^2}{3q-6}$$

Solution:

$$= \frac{5p+2}{6} \cdot \frac{5}{15p+6}$$

$$= \frac{5p+2}{6} \cdot \frac{5}{3(5p+2)}$$

$$= \frac{5}{18}$$

$$= \frac{q^2+2q}{5+q} \cdot \frac{3q-6}{4-q^2}$$

$$= \frac{q(q+2)}{5+q} \cdot \frac{3(q-2)}{(2-q)(2+q)}$$

$$= \frac{q(q+2)3(q-2)}{(-1)(5+q)(q-2)(q+2)}$$

$$= -\frac{3q}{5+q}$$

7.2 Adding and Subtracting Rational Expressions

Objectives

1. Add and subtract rational expressions with the same denominator.
2. Find a least common denominator.
3. Add and subtract rational expressions with different denominators.

PEARSON

Objective 1

Add and subtract rational expressions with the same denominator.

Add and subtract rational expressions with the same denominator.

Adding or Subtracting Rational Expressions

Step 1 **If the denominators are the same,** add or subtract the numerators. Place the result over the common denominator.

If the denominators are different, first find the least common denominator. Write all rational expressions with this least common denominator, and then add or subtract the numerators. Place the result over the common denominator.

Step 2 **Simplify.** Write all answers in lowest terms.

CLASSROOM EXAMPLE 1 **Adding and Subtracting Rational Expressions Same Denominator**

Add or subtract as indicated.

Solution:

$$\frac{7x}{9}+\frac{y}{9} \quad =\frac{7x+y}{9}$$

$$\frac{8}{3x^3}-\frac{14}{3x^3} \quad =\frac{8-14}{3x^3} \quad =\frac{-6}{3x^3} \quad =\frac{-2}{x^3}$$

$$\frac{r}{r^2-t^2}+\frac{t}{r^2-t^2} = \frac{r+t}{r^2-t^2} \quad = \frac{r+t}{(r+t)(r-t)} \quad = \frac{1}{r-t}$$

$$\frac{6}{x^2+3x-18}+\frac{x}{x^2+3x-18} \quad = \frac{6+x}{x^2+3x-18}$$

$$= \frac{6+x}{(x+6)(x-3)} \quad = \frac{1}{x-3}$$

Objective 2

Find a least common denominator.

Find a least common denominator.

Finding the Least Common Denominator

Step 1 **Factor** each denominator.

Step 2 **Find the least common denominator.** The LCD is the product of all of the different factors from each denominator, with each factor raised to the ***greatest*** power that occurs in any denominator.

CLASSROOM EXAMPLE 2 **Finding Least Common Denominators**

Find the LCD for each group of denominators.

$10a^3b^5, 15a^2b^6$

Solution:

Factor. $10a^3b^5 = 2 \cdot 5 \cdot a^3 \cdot b^5$

$15a^2b^6 = 3 \cdot 5 \cdot a^2 \cdot b^6$

LCD $= 2 \cdot 3 \cdot 5 \cdot a^3 \cdot b^6 = 30a^3b^6$

$z, z + 6$

Each denominator is already factored.

LCD = $z(z + 6)$

CLASSROOM EXAMPLE 2 **Finding Least Common Denominators (cont'd)**

Find the LCD for each group of denominators.

$m^2 - 16$, $m^2 + 8m + 16$

Solution:

Factor. $m^2 - 16 = (m + 4)(m - 4)$

$m^2 + 8m + 16 = (m + 4)^2$

LCD $= (m + 4)^2(m - 4)$

$x^2 - 2x + 1$, $x^2 - 4x + 3$, $4x - 4$

$x^2 - 2x + 1 = (x - 1)(x - 1)$

$x^2 - 4x + 3 = (x - 1)(x - 3)$

$4x - 4 = 4(x - 1)$

LCD $= 4(x - 1)^2(x - 3)$

Objective 3

Add and subtract rational expressions with different denominators.

CLASSROOM EXAMPLE 3 **Adding and Subtracting Rational Expressions (Different Denominators)**

Add or subtract as indicated.

Solution:

$$\frac{6}{m} + \frac{1}{4m} = \frac{6 \cdot 4}{m \cdot 4} + \frac{1}{4m} = \frac{24}{4m} + \frac{1}{4m} = \frac{24 + 1}{4m} = \frac{25}{4m}$$

$$\frac{2}{y} - \frac{1}{y + 4} = \frac{2(y + 4)}{y(y + 4)} - \frac{(1)y}{(y + 4)y} = \frac{2(y + 4)}{y(y + 4)} - \frac{y}{y(y + 4)}$$

$$= \frac{2y + 8 - y}{y(y + 4)} = \frac{y + 8}{y(y + 4)}$$

CLASSROOM EXAMPLE 4 **Subtracting Rational Expressions**

Subtract.

$$\frac{5x + 7}{2x + 7} - \frac{-x - 14}{2x + 7}$$

Solution:

The denominators are already the same for both rational expressions. The subtraction sign must be applied to both terms in the numerator of the second rational expression.

$$= \frac{5x + 7 - (-x - 14)}{2x + 7} = \frac{5x + 7 + x + 14}{2x + 7} = \frac{6x + 21}{2x + 7}$$

$$= \frac{3(2x + 7)}{2x + 7} = 3$$

CLASSROOM EXAMPLE 4 **Subtracting Rational Expressions (cont'd)**

Subtract.

$$\frac{2}{r - 2} - \frac{r + 3}{r - 1}$$

Solution:

The LCD is $(r - 2)(r - 1)$.

$$= \frac{2(r - 1)}{(r - 2)(r - 1)} - \frac{(r + 3)(r - 2)}{(r - 1)(r - 2)} = \frac{2r - 2 - (r^2 + r - 6)}{(r - 2)(r - 1)}$$

$$= \frac{2r - 2 - r^2 - r + 6}{(r - 2)(r - 1)} = \frac{-r^2 + r + 4}{(r - 2)(r - 1)}$$

CLASSROOM EXAMPLE 5

Adding and Subtracting Rational Expressions (Denominators Are Opposites)

Add.

$$\frac{2}{x-3}+\frac{1}{3-x}.$$

Solution:

To get a common denominator of $x - 3$, multiply both the numerator and denominator of the second expression by -1.

$$=\frac{2}{x-3}+\frac{1(-1)}{(3-x)(-1)} \quad =\frac{2}{x-3}+\frac{-1}{x-3} \quad =\frac{2+(-1)}{x-3} \quad =\frac{1}{x-3}$$

CLASSROOM EXAMPLE 6

Adding and Subtracting Three Rational Expressions

Add and subtract as indicated.

$$\frac{4}{x-5}+\frac{-2}{x}-\frac{10}{x^2-5x}$$

Solution:

$$=\frac{4}{x-5}+\frac{-2}{x}-\frac{10}{x(x-5)}$$

$$=\frac{4x}{(x-5)x}+\frac{-2(x-5)}{x(x-5)}-\frac{10}{x(x-5)}$$

$$=\frac{4x+(-2)(x-5)-10}{x(x-5)}$$

$$=\frac{4x+-2x+10-10}{x(x-5)} \quad =\frac{2x}{x(x-5)} \quad =\frac{2}{x-5}$$

CLASSROOM EXAMPLE 7

Subtracting Rational Expressions

Subtract.

$$\frac{-a}{a^2+3a-4}-\frac{4a}{a^2+7a+12}$$

Solution: $$=\frac{-a}{(a+4)(a-1)}-\frac{4a}{(a+4)(a+3)}$$

LCD is $(a+4)(a-1)(a+3).$

$$=\frac{-a(a+3)}{(a+4)(a-1)(a+3)}-\frac{4a(a-1)}{(a+4)(a-1)(a+3)}$$

CLASSROOM EXAMPLE 7

Subtracting Rational Expressions (cont'd)

$$=\frac{-a(a+3)-4a(a-1)}{(a+4)(a-1)(a+3)}$$

$$=\frac{-a^2-3a-4a^2+4a}{(a+4)(a-1)(a+3)}$$ **Distributive property**

$$=\frac{-5a^2+a}{(a+4)(a-1)(a+3)}$$ **Combine terms in the numerator.**

CLASSROOM EXAMPLE 8

Adding Rational Expressions

Add.

$$\frac{4}{p^2-6p+9}+\frac{1}{p^2+2p-15}$$

Solution: $$=\frac{4}{(p-3)(p-3)}+\frac{1}{(p+5)(p-3)}$$

LCD is $(p-3)^2(p+5).$

$$=\frac{4(p+5)}{(p-3)^2(p+5)}+\frac{1(p-3)}{(p+5)(p-3)^2}$$

CLASSROOM EXAMPLE 8

Adding Rational Expressions (cont'd)

$$=\frac{4(p+5)+1(p-3)}{(p-3)^2(p+5)}$$

$$=\frac{4p+20+p-3}{(p-3)^2(p+5)}$$ **Distributive property**

$$=\frac{5p+17}{(p-3)^2(p+5)}$$ **Combine terms in the numerator.**

7.3 Complex Fractions

Objectives

1. Simplify complex fractions by simplifying the numerator and denominator (Method 1).
2. Simplify complex fractions by multiplying by a common denominator (Method 2).
3. Compare the two methods of simplifying complex fractions.
4. Simplify rational expressions with negative exponents.

PEARSON

Complex Fractions

A **complex fraction** is a quotient having a fraction in the numerator, denominator, or both.

$$\frac{1+\frac{1}{x}}{2}, \quad \frac{\frac{4}{y}}{6-\frac{3}{y}}, \quad \text{and} \quad \frac{\frac{m^2-9}{m+1}}{\frac{m+3}{m^2-1}}.$$

Objective 1

Simplify complex fractions by simplifying the numerator and denominator (Method 1).

Simplify complex fractions by simplifying the numerator and denominator (Method 1).

Simplifying a Complex Fraction: Method 1

Step 1 Simplify the numerator and denominator separately.

Step 2 Divide by multiplying the numerator by the reciprocal of the denominator.

Step 3 Simplify the resulting fraction if possible.

CLASSROOM EXAMPLE 1 **Simplifying Complex Fractions (Method 1)**

Use Method 1 to simplify the complex fraction.

$$\frac{\frac{y+2}{y}}{\frac{y-2}{3y}}$$

Both the numerator and denominator are already simplified.

Solution:

$$=\frac{y+2}{y}\div\frac{y-2}{3y}$$ **Write as a division problem.**

$$=\frac{y+2}{y}\cdot\frac{3y}{y-2}$$ **Multiply by the reciprocal.**

$$=\frac{3(y+2)}{y-2}$$ **Multiply.**

CLASSROOM EXAMPLE 1 **Simplifying Complex Fractions (Method 1) (cont'd)**

Use Method 1 to simplify the complex fraction.

$$\frac{4-\frac{3}{x}}{5-\frac{1}{x}}$$

Simplify the numerator and denominator.

Solution:

$$=\frac{4\left(\frac{x}{x}\right)-\frac{3}{x}}{5\left(\frac{x}{x}\right)-\frac{1}{x}}$$

$$=\frac{\frac{4x}{x}-\frac{3}{x}}{\frac{5x}{x}-\frac{1}{x}}$$

$$=\frac{\frac{4x-3}{x}}{\frac{5x-1}{x}}$$

$$=\frac{4x-3}{x}\div\frac{5x-1}{x}$$

$$=\frac{4x-3}{x}\cdot\frac{x}{5x-1} \quad =\frac{4x-3}{5x-1}$$

Simplify complex fractions by multiplying by a common denominator (Method 2).

Simplifying a Complex Fraction: Method 2

Step 1 Multiply the numerator and denominator of the complex fraction by the least common denominator of the fractions in the numerator and the fractions in the denominator of the complex fraction.

Step 2 Simplify the resulting fraction if possible.

CLASSROOM EXAMPLE 2 **Simplifying Complex Fractions (Method 2)**

Use Method 2 to simplify the complex fraction.

$$\frac{4-\frac{3}{x}}{5-\frac{1}{x}}$$

The LCD is *x*. Multiply the numerator and denominator by *x*.

Solution:

$$=\frac{\left(4-\frac{3}{x}\right)\cdot x}{\left(5-\frac{1}{x}\right)\cdot x} \qquad =\frac{4\cdot x-\frac{3}{x}\cdot x}{5\cdot x-\frac{1}{x}\cdot x} \qquad =\frac{4x-3}{5x-1}$$

CLASSROOM EXAMPLE 2 **Simplifying Complex Fractions (Method 2) (cont'd)**

Use Method 2 to simplify the complex fraction.

$$\frac{3y+\frac{4}{y+1}}{2y-\frac{3}{y}}$$

Multiply the numerator and denominator by the LCD *y*(*y* + 1).

Solution:

$$=\frac{\left(3y+\frac{4}{y+1}\right)\cdot y(y+1)}{\left(2y-\frac{3}{y}\right)\cdot y(y+1)} \qquad =\frac{3y[y(y+1)]+\frac{4}{y+1}\cdot y(y+1)}{2y[y(y+1)]-\frac{3}{y}\cdot y(y+1)}$$

$$=\frac{3y^2(y+1)+4y}{2y^2(y+1)-3(y+1)} \qquad =\frac{3y^3+3y^2+4y}{2y^3+2y^2-3y-3}$$

Objective 3

Compare the two methods of simplifying complex fractions.

CLASSROOM EXAMPLE 3 **Simplifying Complex Fractions (Both Methods)**

Simplify the complex fraction by both methods.

$$\frac{\frac{5}{y+2}}{\frac{-3}{y^2-4}}$$

Solution:

Method 1

$$=\frac{\frac{5}{y+2}}{\frac{-3}{(y-2)(y+2)}}$$

$$=\frac{5}{y+2}\div\frac{-3}{(y+2)(y-2)}$$

$$=\frac{5}{y+2}\cdot\frac{(y+2)(y-2)}{-3}$$

$$=\frac{5(y-2)}{-3}$$

Method 2

$$=\frac{\frac{5}{y+2}}{\frac{-3}{(y-2)(y+2)}}$$

$$=\frac{\frac{5}{y+2}\cdot(y+2)(y-2)}{\frac{-3}{(y-2)(y+2)}\cdot(y+2)(y-2)}$$

$$=\frac{5(y-2)}{-3}$$

CLASSROOM EXAMPLE 3 **Simplifying Complex Fractions (Both Methods) (cont'd)**

Simplify the complex fraction by both methods.

$$\frac{\frac{1}{a}-\frac{1}{b}}{\frac{1}{a^2}-\frac{1}{b^2}}$$

Solution: **Method 1**

$$=\frac{\frac{b}{ab}-\frac{a}{ab}}{\frac{b^2}{a^2b^2}-\frac{a^2}{a^2b^2}}$$

LCD = *ab*

LCD = *a*²*b*²

$$=\frac{\frac{b-a}{ab}}{\frac{b^2-a^2}{a^2b^2}}$$

$$=\frac{\frac{b-a}{ab}}{\frac{(b+a)(b-a)}{a^2b^2}}$$

$$=\frac{b-a}{ab}\div\frac{(b+a)(b-a)}{a^2b^2}$$

$$=\frac{b-a}{ab}\cdot\frac{a^2b^2}{(b+a)(b-a)}$$

$$=\frac{ab}{b+a}$$

CLASSROOM EXAMPLE 3 — **Simplifying Complex Fractions (Both Methods) (cont'd)**

Simplify the complex fraction by both methods.

$$\frac{\frac{1}{a}-\frac{1}{b}}{\frac{1}{a^2}-\frac{1}{b^2}}$$

Solution: Method 2

LCD of the numerator and denominator is a^2b^2.

$$=\frac{\left(\frac{1}{a}-\frac{1}{b}\right)\cdot a^2b^2}{\left(\frac{1}{a^2}-\frac{1}{b^2}\right)\cdot a^2b^2} \qquad =\frac{ab^2-a^2b}{b^2-a^2}$$

$$=\frac{ab(b-a)}{(b+a)(b-a)} \qquad =\frac{ab}{b+a}$$

CLASSROOM EXAMPLE 4 — **Simplifying Rational Expressions with Negative Exponents**

Simplify the expression, using only positive exponents in the answer.

$$\frac{a^{-2}+b^{-1}}{a^{-1}-5b^{-3}}$$

Solution:

$$=\frac{\frac{1}{a^2}+\frac{1}{b}}{\frac{1}{a}-\frac{5}{b^3}}$$

LCD = a^2b^3

$$=\frac{a^2b^3\left(\frac{1}{a^2}+\frac{1}{b}\right)}{a^2b^3\left(\frac{1}{a}-\frac{5}{b^3}\right)}$$

$$=\frac{a^2b^3\cdot\frac{1}{a^2}+a^2b^3\cdot\frac{1}{b}}{a^2b^3\cdot\frac{1}{a}-a^2b^3\cdot\frac{5}{b^3}} \qquad =\frac{b^3+a^2b^2}{ab^3-5a^2}$$

CLASSROOM EXAMPLE 4 — **Simplifying Rational Expressions with Negative Exponents (cont'd)**

Simplify the expression, using only positive exponents in the answer.

$$\frac{x^{-3}+2y^{-1}}{y+2x^3}$$

Solution:

$$=\frac{\frac{1}{x^3}+\frac{2}{y}}{y+2x^3}$$

Write with positive exponents.

$$=\frac{\frac{1\cdot y}{x^3\cdot y}+\frac{2\cdot x^3}{y\cdot x^3}}{y+2x^3}$$

LCD = x^3y

$$=\frac{\frac{y+2x^3}{x^3y}}{\frac{y+2x^3}{1}}$$

$$=\frac{y+2x^3}{x^3y}\div\frac{y+2x^3}{1}$$

$$=\frac{y+2x^3}{x^3y}\cdot\frac{1}{y+2x^3}$$

$$=\frac{1}{x^3y}$$

7.4 Equations with Rational Expressions and Graphs

Objectives

1 Determine the domain of the variable in a rational equation.

2 Solve rational equations.

3 Recognize the graph of a rational function.

PEARSON

Determine the domain of the variable in a rational equation.

A **rational equation** is an equation that contains at least one rational expression with a variable in the denominator.

The **domain of the variable in a rational equation** is the intersection of the domains of the rational expressions in the equation.

CLASSROOM EXAMPLE 1 **Determining the Domains of the Variables in Rational Equations**

Find the domain of the variable in each equation.

Solution:

$$\frac{3}{x}+\frac{1}{4}=\frac{9}{4x}$$

The domains of the three rational expressions are, $\{x|x \neq 0\}$, $(-\infty, \infty)$, and $\{x|x \neq 0\}$. The intersection of these three domains is all real numbers except 0, which may be written $\{x|x \neq 0\}$.

$$\frac{2}{x^2-4}+\frac{1}{x+2}=\frac{1}{x-2}$$

The domains of the three rational expressions are, $\{x|x \neq \pm 2\}$, $\{x|x \neq 2\}$, $\{x|x \neq -2\}$. The domain is all real numbers except 2 and -2, $\{x|x \neq \pm 2\}$.

Objective 2

Solve rational equations.

Solve rational expressions.

To solve rational equations, we multiply all terms in the equation by the LCD to clear the fractions. ***We can do this only with equations, not expressions.***

Solving an Equation with Rational Expressions

Step 1 **Determine the domain of the variable.**

Step 2 **Multiply each side of the equation by the LCD** to clear the fractions.

Step 3 **Solve** the resulting equation.

Step 4 **Check** that each proposed solution is in the domain, and discard any values that are not. Check the remaining proposed solution(s) in the original equation.

CLASSROOM EXAMPLE 2 **Solving a Rational Equation**

Solve.

$$-\frac{3}{20}+\frac{2}{x}=\frac{5}{4x}$$

The domain, excludes 0.

Solution:

$$20x\left(-\frac{3}{20}+\frac{2}{x}\right)=20x\left(\frac{5}{4x}\right)$$ **Multiply by the LCD, 20*x*.**

$$20x\left(-\frac{3}{20}\right)+20x\left(\frac{2}{x}\right)=20x\left(\frac{5}{4x}\right)$$

$$-3x+40=25$$

$$-3x=-15$$

$$x=5 \leftarrow$$ **Proposed solution**

CLASSROOM EXAMPLE 2 — **Solving a Rational Equation (cont'd)**

Check.

$$-\frac{3}{20}+\frac{2}{x}=\frac{5}{4x}$$
$$-\frac{3}{20}+\frac{2}{5}=\frac{5}{4(5)}$$
$$-\frac{3}{20}+\frac{2}{5}=\frac{5}{20}$$
$$-\frac{3}{20}+\frac{8}{20}=\frac{5}{20}$$
$$\frac{5}{20}=\frac{5}{20}$$

The solution set is {5}.

CAUTION When each side of an equation is multiplied by a ***variable*** expression, the resulting "solutions" may not satisfy the original equation. ***You must either determine and observe the domain or check all proposed solutions in the original equation. It is wise to do both.***

CLASSROOM EXAMPLE 3 — **Solving a Rational Equation with No Solution**

Solve.

$$\frac{3}{x+1}=\frac{1}{x-1}-\frac{2}{x^2-1}$$

Solution:

The domain, excludes ±1. The LCD is $(x+1)(x-1)$.

$$(x+1)(x-1)\left(\frac{3}{x+1}\right)=(x+1)(x-1)\left(\frac{1}{x-1}-\frac{2}{x^2-1}\right)$$
$$3(x-1)=x+1-2$$
$$3x-3=x-1$$
$$2x=2$$
$$x=1 \longleftarrow \textbf{Proposed solution}$$

CLASSROOM EXAMPLE 3 — **Solving a Rational Equation with No Solution (cont'd)**

Since the proposed solution is not in the domain, it cannot be an actual solution of the equation. Substituting 1 into the original equation shows why.

$$\frac{3}{x+1}=\frac{1}{x-1}-\frac{2}{x^2-1}$$
$$\frac{3}{1+1}=\frac{1}{1-1}-\frac{2}{1^2-1}$$
$$\frac{3}{2}=\frac{1}{0}-\frac{2}{0}$$

Division by 0 is undefined. The equation has no solution and the solution set is ∅.

CLASSROOM EXAMPLE 4 — **Solving a Rational Equation**

Solve.

$$\frac{4}{x^2+x-6}-\frac{1}{x^2-4}=\frac{2}{x^2+5x+6}$$

Solution:

Factor each denominator.

$x^2 + x - 6 = (x + 3)(x - 2)$, so $x \neq -3, 2$.

$x^2 - 4 = (x + 2)(x - 2)$, so $x \neq \pm 2$.

$x^2 + 5x + 6 = (x + 3)(x + 2)$, so $x \neq -3, -2$.

The domain is $\{x \mid x \neq -3, \pm 2\}$.

The LCD = $(x + 3)(x + 2)(x - 2)$.

CLASSROOM EXAMPLE 4 — **Solving a Rational Equation (cont'd)**

$$(x+3)(x+2)(x-2)\left[\frac{4}{x^2+x-6}-\frac{1}{x^2-4}\right]=(x+3)(x+2)(x-2)\frac{2}{x^2+5x+6}$$
$$4(x+2)-(x+3)=2(x-2)$$
$$4x+8-x-3=2x-4$$
$$3x+5=2x-4$$
$$x=-9 \longleftarrow \textbf{Proposed solution}$$

The solution checks in the original equation. The solution set is {–9}.

CLASSROOM EXAMPLE 5 — **Solving a Rational Equation**

Solve.

$$\frac{2}{x+3}-\frac{1}{x-1}=\frac{-x^2-3x}{x^2+2x-3}$$

Solution:

Factor each denominator to find the LCD.

$x + 3 = 0$, so $x \neq -3$.

$x - 1 = 0$, so $x \neq 1$.

$x^2 + 2x - 3 = (x + 3)(x - 1)$, so $x \neq -3, 1$.

The domain is $\{x \mid x \neq -3, 1\}$.

The LCD = $(x + 3)(x - 1)$.

CLASSROOM EXAMPLE 5 **Solving a Rational Equation (cont'd)**

$$(x+3)(x-1)\left(\frac{2}{x+3}-\frac{1}{x-1}\right)=(x+3)(x-1)\left(\frac{-x^2-3x}{x^2+2x-3}\right)$$

$$2(x-1)-1(x+3)=-x^2-3x$$
$$2x-2-x-3=-x^2-3x$$
$$x-5=-x^2-3x$$
$$x^2+4x-5=0$$
$$(x+5)(x-1)=0$$
$$x+5=0 \quad \text{or} \quad x-1=0$$
$$x=-5 \quad \text{or} \quad x=1$$

CLASSROOM EXAMPLE 5 **Solving a Rational Equation (cont'd)**

$$\frac{2}{x+3}-\frac{1}{x-1}=\frac{-x^2-3x}{x^2+2x-3}$$
$$x=-5 \quad \text{or} \quad x=1$$

Because 1 is not in the domain of the equation, it is not a solution.

$$\frac{2}{-5+3}-\frac{1}{-5-1}=\frac{-5^2-3(-5)}{(-5)^2+2(-5)-3}$$
$$\frac{2}{-2}-\frac{1}{-6}=\frac{-25+15}{25-10-3}$$
$$-1+\frac{1}{6}=-\frac{10}{12} \qquad -\frac{5}{6}=-\frac{5}{6}$$

The solution set is {–5}.

Objective 3

Recognize the graph of a rational function.

Recognize the graph of a rational function.

A function defined by a quotient of polynomials is a **rational function.** Because one or more values of *x* may be excluded from the domain of a rational function, their graphs are often **discontinuous.** That is, there will be one or more breaks in the graph.

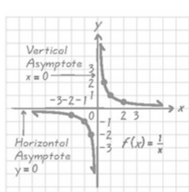
One simple rational function, defined by $f(x)=\frac{1}{x}$ and graphed on the next slide, is the **reciprocal function.** The domain of this function includes all real numbers except 0. Thus, this function pairs every real number except 0 with its reciprocal.

Since the domain of this function includes all real numbers except 0, there is no point on the graph with *x* = 0. The vertical line with equation *x* = 0 is called a **vertical asymptote** of the graph. Also, the horizontal line with equation *y* = 0 is called a **horizontal asymptote.**

Recognize the graph of a rational function.

The closer negative values of *x* are to 0, the smaller ("more negative") *y* is. The closer positive values of *x* are to 0, the larger *y* is.

x	−3	−2	−1	−0.5	−0.25	−0.1	0.1	0.25	0.5	1	2	3
y	$-\frac{1}{3}$	$-\frac{1}{2}$	−1	−2	−4	−10	10	4	2	1	$\frac{1}{2}$	$\frac{1}{3}$



Reciprocal function

$f(x)=\frac{1}{x}$

Domain: $\{x \mid x \neq 0\}$
Range: $\{x \mid x \neq 0\}$

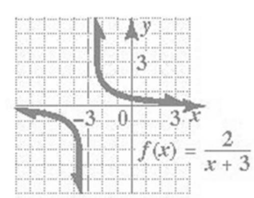
In general, if the *y*-values of a rational function approach ∞ or −∞ as the *x*-values approach a real number *a*, the vertical line *x* = *a* is a vertical asymptote of the graph. Also, if the *x*-values approach a real number *b* as $|x|$ increases without bound, the horizontal line *y* = *b* is a horizontal asymptote of the graph.

CLASSROOM EXAMPLE 6 **Graphing a Rational Function**

Graph, and give the equations of the vertical and horizontal asymptotes.

$$f(x)=\frac{2}{x+3}$$

Solution:



The vertical asymptote: *x* = − 3
The horizontal asymptote: *y* = 0

7.5 Applications of Rational Expressions

Objectives

1. Find the value of an unknown variable in a formula.
2. Solve a formula for a specified variable.
3. Solve applications by using proportions.
4. Solve applications about distance, rate, and time.
5. Solve applications about work rates.

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Objective 1

Find the value of an unknown variable in a formula.

CLASSROOM EXAMPLE 1 Finding the Value of a Variable in a Formula

Use the formula $\frac{1}{f}=\frac{1}{p}+\frac{1}{q}$ to find p if f = 15 cm and q = 25 cm.

Solution:

$$\frac{1}{f}=\frac{1}{p}+\frac{1}{q}$$

$$\frac{1}{15}=\frac{1}{p}+\frac{1}{25}$$ **Let f = 15 and q = 25.**

$$75p\cdot\frac{1}{15}=75p\left(\frac{1}{p}+\frac{1}{25}\right)$$ **Multiply by the LCD, 75*p*.**

$$5p=75+3p$$

$$2p=75$$

$$p=\frac{75}{2}$$

Objective 2

Solve a formula for a specified variable.

CLASSROOM EXAMPLE 2 Finding a Formula for a Specified Variable

Solve $\frac{3}{p}+\frac{3}{q}=\frac{5}{r}$ for q.

Solution:

$$pqr\left(\frac{3}{p}+\frac{3}{q}\right)=pqr\cdot\frac{5}{r}$$ **Multiply by the LCD, *pqr*.**

$$3qr+3pr=5pq$$ **Distributive property.**

$$3pr=5pq-3qr$$ **Subtract 3*qr* to get all *q* terms on same side of equation.**

$$3pr=q(5p-3r)$$ **Factor out *q*.**

$$\frac{3pr}{5p-3r}=q \quad \text{or} \quad q=\frac{3pr}{5p-3r}$$ **Divide.**

CLASSROOM EXAMPLE 3 Solving a Formula for a Specified Variable

Solve $A=\frac{Rr}{R+r}$ for R.

Solution:

$$(R+r)A=(R+r)\left(\frac{Rr}{R+r}\right)$$

$$A(R+r)=Rr$$

$$AR+Ar=Rr$$

$$AR-Rr=-Ar$$

$$R(A-r)=-Ar$$

$$R=\frac{-Ar}{A-r} \quad \text{or} \quad \frac{Ar}{r-A}$$

Solve applications by using proportions.

A **ratio** is a comparison of two quantities. The ratio of *a* to *b* may be written in any of the following ways:

$$a \text{ to } b, \qquad a:b, \qquad \text{or} \qquad \frac{a}{b}.$$

Ratio of *a* to *b*

Ratios are usually written as quotients in algebra. A **proportion** is a statement that two ratios are equal, such as

$$\frac{a}{b} = \frac{c}{d}.$$

Proportion

CLASSROOM EXAMPLE 4 **Solving a Proportion**

In 2008, approximately 9.9% (that is, 9.9 of every 100) of the 74,510,000 children under 18 yr of age in the United States had no health insurance. How many such children were uninsured? (*Source:* U.S. Census Bureau.)

Solution:

Step 1 **Read** the problem.

Step 2 **Assign a variable.**
Let x = the number (in millions) who had no health insurance.

Step 3 **Write an equation.** To get an equation, set up a proportion.

$$\frac{9.9}{100} = \frac{x}{74{,}510{,}000}$$

CLASSROOM EXAMPLE 4 **Solving a Proportion (cont'd)**

Step 4 **Solve.**

$$\frac{9.9}{100} = \frac{x}{74{,}510{,}000}$$

$$74{,}510{,}000 \cdot \frac{9.9}{100} = 74{,}510{,}000 \cdot \frac{x}{74{,}510{,}000}$$

$$745{,}100 \cdot 9.9 = x$$

$$x = 7{,}376{,}490$$

Step 5 **State the answer**. There were 7,376,490 children under 18 years of age in the United States with no health insurance in 2008.

Step 6 **Check**. The ratio $\frac{7{,}376{,}490}{74{,}510{,}000} = \frac{9.9}{100}.$

CLASSROOM EXAMPLE 5 **Solving a Proportion Involving Rates**

Lauren's car uses 15 gal of gasoline to drive 390 mi. She has 6 gal of gasoline in the car, and she wants to know how much more gasoline she will need to drive 800 mi. If we assume that the car continues to use gasoline at the same rate, how many more gallons will she need?

Solution:

Step 1 **Read** the problem.

Step 2 **Assign a variable.**
Let x = the additional number of gallons needed.

CLASSROOM EXAMPLE 5 **Solving a Proportion Involving Rates (cont'd)**

Step 3 **Write an equation.** She knows that she can drive 390 miles with 15 gallons of gasoline. She wants to drive 800 miles using (6 + x) gallons of gasoline. Set up a proportion.

Step 4 **Solve.**

$$\frac{390}{15} = \frac{800}{6+x}$$

$$\frac{26}{1} = \frac{800}{6+x} \qquad \textbf{Reduce.}$$

$$26(6+x) = 800$$

$$156 + 26x = 800$$

$$26x = 644$$

$$x = \frac{644}{26}$$

$$x \approx 24.8$$

CLASSROOM EXAMPLE 5 **Solving a Proportion Involving Rates (cont'd)**

Step 5 **State the answer**. She will need about 24.8 more gallons of gasoline.

Step 6 **Check**. The 6 gallons + 24.8 gallons equals 30.8 gallons. Check the rates (miles/gallon).

$$\frac{390}{15} = 26 \text{ mpg} \qquad \frac{800}{30.8} \approx 26 \text{ mpg}$$

The rates are approximately equal, so the solution is correct.

CLASSROOM EXAMPLE 6 **Solving a Problem about Distance, Rate, and Time**

A plane travels 100 mi against the wind in the same time that it takes to travel 120 mi with the wind. The wind speed is 20 mph. Find the speed of the plane in still air.

Solution:

Step 1 **Read** the problem.

We must find the speed of the plane in still air.

Step 2 **Assign a variable.**

Let x = the speed of the plane in still air.
Use $d = rt$, to complete the table (next slide).

CLASSROOM EXAMPLE 6 **Solving a Problem about Distance, Rate, and Time (cont'd)**

	d	***r***	***t***
Against Wind	100	$x - 20$	$\frac{100}{x-20}$
With Wind	120	$x + 20$	$\frac{100}{x+20}$

Step 3 **Write an equation.** Since the time against the wind equals the time with the wind, we set up this equation.

$$\frac{100}{x-20} = \frac{120}{x+20}$$

CLASSROOM EXAMPLE 6 **Solving a Problem about Distance, Rate, and Time (cont'd)**

Step 4 **Solve.**

Multiply by the LCD $(x - 20)(x + 20)$.

$$\frac{100}{x-20} = \frac{120}{x+20}$$

$$(x-20)(x+20)\frac{100}{x-20} = (x-20)(x+20)\frac{120}{x+20}$$

$$100(x+20) = 120(x-20)$$

$$100x + 2000 = 120x - 2400$$

$$4400 = 20x$$

$$220 = x$$

CLASSROOM EXAMPLE 6 **Solving a Problem about Distance, Rate, and Time (cont'd)**

Step 5 **State the answer.**

The speed of the airplane is 220 mph in still air.

Step 6 **Check.**

$$\frac{100}{220-20} = \frac{120}{220+20}$$

$$\frac{100}{200} = \frac{120}{240}$$

$$\frac{1}{2} = \frac{1}{2}$$

CLASSROOM EXAMPLE 7 **Solving a Problem about Distance, Rate, and Time**

Dona Kenly drove 300 mi north from San Antonio, mostly on the freeway. She usually averaged 55 mph, but an accident slowed her speed through Dallas to 15 mph. If her trip took 6 hr, how many miles did she drive at the reduced rate?

Solution:

Step 1 **Read** the problem.

We must find how many miles she drove at the reduced speed.

Step 2 **Assign a variable.**

Let x = the distance at reduced speed.
Use $d = rt$, to complete the table (next slide).

CLASSROOM EXAMPLE 7 **Solving a Problem about Distance, Rate, and Time (cont'd)**

	d	***r***	***t***
Normal Speed	$300 - x$	55	$\frac{300-x}{55}$
Reduced Speed	x	15	$\frac{x}{15}$

Step 3 **Write an equation.**

Time on freeway	plus	Time at reduced speed	equals 6 hr.
$\frac{300-x}{55}$	$+$	$\frac{x}{15}$	$= 6$

CLASSROOM EXAMPLE 7 **Solving a Problem about Distance, Rate, and Time (cont'd)**

Step 4 **Solve.**

Multiply by the LCD, 165.

$$165\left(\frac{300-x}{55}+\frac{x}{15}\right)=165\cdot 6$$
$$3(300-x)+11x=990$$
$$900-3x+11x=990$$
$$8x=90$$
$$x=\frac{90}{8}\text{ or }11\tfrac{1}{4}$$

Step 5 **State the answer.** She drove 11 ¼ miles at reduced speed.

Step 6 **Check.** The check is left to the student.

Solve applications about work rates.

Objective 5

PROBLEM-SOLVING HINT

People work at different rates. If the letters *r, t,* and *A* represent the rate at which work is done, the time required, and the amount of work accomplished, respectively, then *A* = *rt*. Notice the similarity to the distance formula, *d* = *rt.*

Amount of work can be measured in terms of jobs accomplished. Thus, if 1 job is completed, then *A* = 1, and the formula gives the rate as

$$1=rt,\quad\text{or}\quad r=\frac{1}{t}.$$

Solve applications about work rates.

Rate of Work

If a job can be accomplished in *t* units of time, then the rate of work is

$$\frac{1}{t}\text{ job per unit of time.}$$

CLASSROOM EXAMPLE 8 **Solving a Problem about Work**

Stan needs 45 minutes to do the dishes, while Bobbie can do them in 30 minutes. How long will it take them if they work together?

Solution:

Step 1 **Read** the problem.

We must determine how long it will take them working together to wash the dishes.

Step 2 **Assign a variable.**

Let *x* = the time it will take them working together.

	Rate	Time Working Together	Fractional Part of the Job Done
Stan	$\frac{1}{45}$	*x*	$\frac{1}{45}x$
Bobbie	$\frac{1}{30}$	*x*	$\frac{1}{30}x$

CLASSROOM EXAMPLE 8 **Solving a Problem about Work (cont'd)**

Step 3 **Write an equation.**

Part done by Stan	plus	Part done by Bobbie	equals	1 whole job.
$\frac{1}{45}x$	$+$	$\frac{1}{30}x$	$=$	1

Step 4 **Solve.** Multiply by the LCD, 90.

$$90\left(\frac{1}{45}x+\frac{1}{30}x\right)=90\cdot 1$$
$$2x+3x=90$$
$$5x=90$$
$$x=18$$

CLASSROOM EXAMPLE 8 **Solving a Problem about Work (cont'd)**

Step 5 **State the answer.**

It will take them 18 minutes working together.

Step 6 **Check.**

The check is left to the student.

7.6 Variation

Objectives

1 Write an equation expressing direct variation.

2 Find the constant of variation, and solve direct variation problems.

3 Solve inverse variation problems.

4 Solve joint variation problems.

5 Solve combined variation problems.

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Write an equation expressing direct variation.

Direct Variation

y* varies directly as *x if there exists a real number *k* such that

y = kx.

y is said to be **proportional to** *x*. The number *k* is called the **constant of variation.**

In direct variation, for k > 0, as the value of x increases, the value of y also increases. Similarly, as x decreases, y decreases.

CLASSROOM EXAMPLE 1 **Finding the Constant of Variation and the Variation Equation**

If 7 kg of steak cost $45.50, how much will 1 kg of steak cost?

Solution:

Let *C* represent the cost of *p* kilograms of steak. *C* varies directly as *p*, so *C* = *kp*.

Here *k* represents the cost of one kilogram of steak. Since *C* = 45.50 when *p* =7,

$$45.50 = k \cdot 7.$$

$$k = \frac{45.50}{7}$$

$$k = 6.50$$

One kilogram of steak costs $6.50, and *C* and *p* are related by C = 6.50*p*.

CLASSROOM EXAMPLE 2 **Solving a Direct Variation Problem**

It costs $52 to use 800 kilowatt-hours of electricity. How much will 650 kilowatt-hours cost?

Solution:

Let *c* represent the cost of using *h* kilowatt-hours. Use *c* = *kh* with *c* = 52 and *h* = 800 to find *k*.

$$c = kh$$

$$52 = k(800)$$

$$\frac{52}{800} = k$$

$$\frac{13}{200} = k$$

CLASSROOM EXAMPLE 2 **Solving a Direct Variation Problem (cont'd)**

So $c = \frac{13}{200}h.$

Let *h* = 650. Find *c*.

$$c = \frac{13}{200}(650)$$

$$c = 42.25$$

Thus, 650 kilowatt-hours costs $42.25.

Find the constant of variation, and solve direct variation problems.

Solving a Variation Problem

Step 1 Write the variation equation.

Step 2 Substitute the initial values and solve for *k*.

Step 3 Rewrite the variation equation with the value of *k* from ***Step 2.***

Step 4 Substitute the remaining values, solve for the unknown, and find the required answer.

Find the constant of variation, and solve direct variation problems.

Direct Variation as a Power

y* varies directly as the *n*th power of *x if there exists a real number *k* such that

$$y = kx^n.$$

CLASSROOM EXAMPLE 3 **Solving a Direct Variation Problem**

Suppose *y* varies directly as the cube of *x*, and $y = 24$ when $x = 2$. Find *y* when $x = 4$.

Solution:

Step 1 *y* varies directly as the cube of *x*, so $y = kx^3$.

Step 2 Find the value of *k*.

$y = 24$ when $x = 2$, so

$$24 = k(2)^3$$
$$24 = k(8)$$
$$3 = k$$

Step 3 Thus, $y = 3x^3$.

Step 4 When $x = 4$, $y = 3(4)^3 \quad = 3(64) \quad = 192.$

Solve inverse variation problems.

Inverse Variation

y* varies inversely as *x if there exists a real number *k* such that

$$y = \frac{k}{x}.$$

Also, ***y* varies inversely as the *n*th power of *x*** if there exists a real number *k* such that

$$y = \frac{k}{x^n}.$$

With inverse variation, where k > 0, as one variable increases, the other variable decreases.

CLASSROOM EXAMPLE 4 **Solving an Inverse Variation Problem**

The current in a simple electrical circuit varies inversely as the resistance. If the current is 80 amps when the resistance is 10 ohms, find the current if the resistance is 16 ohms.

Solution:

Let *C* represent the current, and *R* the resistance.
C varies inversely as *R,* so

$$C = \frac{k}{R}, \quad \text{for some constant } k.$$

Since $C = 80$ when $R = 10$,

$$80 = \frac{k}{10}$$
$$800 = k.$$

CLASSROOM EXAMPLE 4 **Solving an Inverse Variation Problem (cont'd)**

Thus $C = \frac{800}{R}.$

When $R = 16,$

$$C = \frac{800}{16} = 50.$$

The current is 50 ampere.

CLASSROOM EXAMPLE 5 **Solve an Inverse Variation Problem**

Suppose *p* varies inversely as the cube of *q* and $p = 100$ when $q = 3$. Find *p*, given that $q = 5$.

Solution:

p varies inversely as the cube of *q*, so $p = \frac{k}{q^3}.$

$p = 100$ when $q = 3$, so $100 = \frac{k}{3^3}$

$$2700 = k$$

Thus, $p = \frac{2700}{q^3}.$ When $q = 5,$

$$p = \frac{2700}{5^3} = \frac{2700}{125} = \frac{108}{5}.$$

Objective 4

Solve joint variation problems.

Solve joint variation problems.

Joint Variation

y* varies jointly as *x* and *z if there exists a real number *k* such that
y = kxz.

CAUTION Note that ***and*** in the expression "*y* varies directly as *x* and *z*" translates as the product *y* = *kxz*. The word ***and*** does not indicate addition here.

CLASSROOM EXAMPLE 6 **Solving a Joint Variation Problem**

If *x* varies jointly as *y* and z^2, and *x* = 231 when *y* = 3 and *z* = 2, find *x* when *y* = 5 and *z* = 4.

Solution:

x varies jointly as *y* and z^2, so $x = kyz^2.$

x = 231 when *y* = 3 and *z* = 2, so $231 = k(3)(2)^2$

$$77 = k(4)$$

$$k = \frac{77}{4}.$$

CLASSROOM EXAMPLE 6 **Solving a Joint Variation Problem (cont'd)**

Thus, $x = \frac{77}{4}yz^2.$

When *y* = 5 and *z* = 4,

$$x = \frac{77}{4}(5)(4)^2 = 77(5)(4) = 1540.$$

CLASSROOM EXAMPLE 7 **Solving a Combined Variation Problem**

Suppose *z* varies jointly as *x* and y^2 and inversely as *w*. Also, $z = \frac{3}{8}$ when *x* = 2, *y* = 3, and *w* = 12. Find *z*, when *x* = 4, *y* = 1, and *w* = 6.

Solution:

z varies jointly as *x* and y^2 and inversely as *w*, so

$$z = \frac{kxy^2}{w}.$$

CLASSROOM EXAMPLE 7 **Solving a Combined Variation Problem (cont'd)**

$z = \frac{3}{8}$ when *x* = 2, *y* = 3 and *w* = 12, so $\frac{3}{8} = \frac{k(2)(3)^2}{12}$

$$\frac{3}{8} = \frac{k(3)}{2}$$

$$k = \frac{3}{8} \cdot \frac{2}{3} = \frac{1}{4}$$

Thus, $z = \frac{\frac{1}{4}xy^2}{w} = \frac{xy^2}{4w}.$

When *x* = 4, *y* = 1, and *w* = 6, $z = \frac{4(1)^2}{4(6)} = \frac{1}{6}.$