8.1 Radical Expressions and Graphs

Objectives

1. Find roots of numbers.
2. Find principal roots.
3. Graph functions defined by radical expressions.
4. Find *n*th roots of *n*th powers.
5. Use a calculator to find roots.

PEARSON

Find roots of numbers.

The opposite (or inverse) of ***squaring*** a number is taking its ***square root.***

$$\sqrt{36} = 6, \text{ because } 6^2 = 36.$$

We now extend our discussion of roots to include ***cube roots*** $\sqrt[3]{}$, ***fourth roots*** $\sqrt[4]{}$, and higher roots.

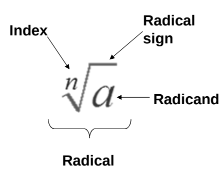
$$\sqrt[n]{a}$$

The *n*th root of *a*, written $\sqrt[n]{a}$, is a number whose *n*th power equals *a*. That is,

$$\sqrt[n]{a} = b \text{ means } b^n = a.$$

Find roots of numbers.

The number *a* is the **radicand.**
n is the **index** or **order**.
The expression is the **radical.**



CLASSROOM EXAMPLE 1 **Simplifying Higher Roots**

Simplify.
Solution:

$\sqrt[3]{27} = 3, \text{ because } 3^3 = 27$

$\sqrt[3]{216} = 6, \text{ because } 6^3 = 216$

$\sqrt[4]{256} = 4, \text{ because } 4^4 = 256$

$\sqrt[5]{243} = 3, \text{ because } 3^5 = 243$

$\sqrt[4]{\frac{16}{81}} = \frac{2}{3}, \text{ because } \left(\frac{2}{3}\right)^4 = \frac{16}{81}$

$\sqrt[3]{0.064} = 0.4, \text{ because } 0.4^3 = 0.064$

Find principal roots.

***n*th Root**

Case 1 If *n* is ***even*** and *a* is ***positive or 0,*** then

$\sqrt[n]{a}$ represents the **principal *n*th root** of *a*,

$-\sqrt[n]{a}$ represents the **negative *n*th root** of *a*.

Case 2 If *n* is ***even*** and *a* is ***negative,*** then

$\sqrt[n]{a}$ is not a real number.

Case 3 If *n* is ***odd***, then

there is exactly one real *n*th root of *a*, written $\sqrt[n]{a}$.

CLASSROOM EXAMPLE 2 **Finding Roots**

Find each root.

Solution:

$\sqrt{36} = 6$

$-\sqrt{36} = -6$

$\sqrt[4]{16} = 2$

$-\sqrt[4]{16} = -2$

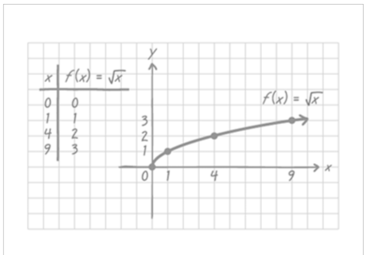
$\sqrt[4]{-16}$ Not a real number.

$\sqrt[5]{243} = 3$

$\sqrt[5]{-243} = -3$

Graph functions defined by radical expressions.

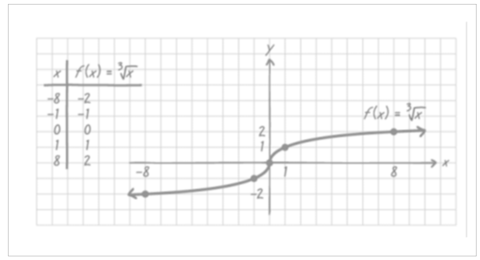
Square Root Function



The domain and range of the square root function are [0, ∞).

Graph functions defined by radical expressions.

Cube Root Function



The domain and range of the cube function are (–∞, ∞).

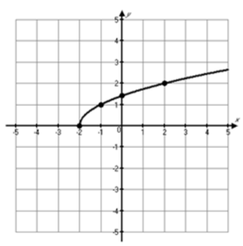
CLASSROOM EXAMPLE 3 **Graphing Functions Defined with Radicals**

Graph the function by creating a table of values. Give the domain and range.

$f(x) = \sqrt{x+2}$

Solution:

x	*f*(*x*)
–2	$\sqrt{-2+2} = 0$
–1	$\sqrt{-1+2} = 1$
0	$\sqrt{0+2} = 1.41$
2	$\sqrt{2+2} = 2$



Domain: [–2, ∞)

Range: [0, ∞)

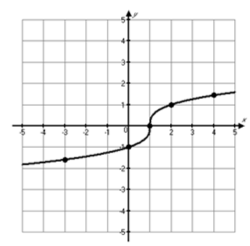
CLASSROOM EXAMPLE 3 **Graphing Functions Defined with Radicals (cont'd)**

Graph the function by creating a table of values. Give the domain and range.

$f(x) = \sqrt[3]{x-1}$

Solution:

X	*f*(*x*)
0	$\sqrt[3]{0-1} = -1$
1	$\sqrt[3]{1-1} = 0$
2	$\sqrt[3]{2-1} = 1$
–3	$\sqrt[3]{-3-1} = -1.587$
4	$\sqrt[3]{4-1} = 1.44$



Domain: (–∞, ∞)

Range: (–∞, ∞)

Objective 4

Find *n*th roots of *n*th powers.

Find *n*th roots of *n*th powers.

$\sqrt{a^2}$

For any real number *a*, $\sqrt{a^2} = |a|$.

That is, the principal square root of a^2 is the absolute value of *a*.

CLASSROOM EXAMPLE 4 **Simplifying Square Roots by Using Absolute Value**

Find each square root.

Solution:

$\sqrt{15^2} = |15| = 15$ $\sqrt{(-12)^2} = |-12| = 12$

$\sqrt{y^2} = |y|$ $\sqrt{(-y^2)} = |-y| = |y|$

Find *n*th roots of *n*th powers.

$$\sqrt[n]{a^n}$$

If *n* is an ***even*** positive integer, then $\sqrt[n]{a^n} = |a|$.

If *n* is an ***odd*** positive integer, then $\sqrt[n]{a^n} = a$.

That is, use absolute value when *n* is even; absolute value is not necessary when *n* is odd.

CLASSROOM EXAMPLE 5 **Simplifying Higher Roots by Using Absolute Value**

Simplify each root.

Solution:

$\sqrt[4]{(-5)^4} = |-5| = 5$

$\sqrt[5]{(-5)^5} = -5$ n is odd

$-\sqrt[6]{(-3)^6} = -|-3| = -3$

$-\sqrt[4]{m^8} = -m^2$ n is even

$\sqrt[3]{x^{24}} = x^8$

$\sqrt[6]{y^{18}} = \sqrt[6]{(y^3)^6} = |y^3|$

Objective 5

Use a calculator to find roots.

CLASSROOM EXAMPLE 6 **Finding Approximations for Roots**

Use a calculator to approximate each radical to three decimal places.

Solution:

$\sqrt{17} = 4.123$ $-\sqrt{362} = -19.026$

$\sqrt[3]{9482} = 21.166$ $\sqrt[4]{6825} = 9.089$

CLASSROOM EXAMPLE 7 **Using Roots to Calculate Resonant Frequency**

In electronics, the resonant frequency *f* of a circuit may be found by the formula $f = \frac{1}{2\pi\sqrt{LC}}$ where *f* is the cycles per second, *L* is in henrys, and *C* is in farads. (Henrys and farads are units of measure in electronics). Find the resonant frequency *f* if $L = 6 \times 10^{-5}$ and $C = 4 \times 10^{-9}$.

Solution:

$$f = \frac{1}{2\pi\sqrt{LC}} \quad f = \frac{1}{2\pi\sqrt{(6\times10^{-5})(4\times10^{-9})}} \approx 324,874$$

About 325,000 cycles per second.