8.3 Simplifying Radical Expressions

Objectives

1. Use the product rule for radicals.
2. Use the quotient rule for radicals.
3. Simplify radicals.
4. Simplify products and quotients of radicals with different indexes.
5. Use the Pythagorean theorem.
6. Use the distance formula.

PEARSON

Use the product rule for radicals.

Product Rule for Radicals

If $\sqrt[n]{a}$ and $\sqrt[n]{b}$ are real numbers and *n* is a natural number, then

$$\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{ab}.$$

That is, the product of two *n*th roots is the *n*th root of the product.

CAUTION *Use the product rule only when the radicals have the same index.*

CLASSROOM EXAMPLE 1 **Using the Product Rule**

Multiply. Assume that all variables represent positive real numbers.

Solution:

$$\sqrt{5} \cdot \sqrt{13} = \sqrt{5 \cdot 13} = \sqrt{65}$$

$$\sqrt{7} \cdot \sqrt{xy} = \sqrt{7xy}$$

CLASSROOM EXAMPLE 2 **Using the Product Rule**

Multiply. Assume that all variables represent positive real numbers.

Solution:

$$\sqrt[3]{2} \cdot \sqrt[3]{7} = \sqrt[3]{2 \cdot 7} = \sqrt[3]{14}$$

$$\sqrt[6]{8r^2} \cdot \sqrt[6]{2r^3} = \sqrt[6]{16r^5}$$

$$\sqrt[5]{9y^2x} \cdot \sqrt[5]{8xy^2} = \sqrt[5]{72y^4x^2}$$

$\sqrt{7} \cdot \sqrt[3]{5}$ This expression cannot be simplified by using the product rule.

Use the quotient rule for radicals.

Quotient Rule for Radicals

If $\sqrt[n]{a}$ and $\sqrt[n]{b}$ are real numbers, $b \neq 0$, and *n* is a natural number, then

$$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}.$$

That is, the *n*th root of a quotient is the quotient of the *n*th roots.

CLASSROOM EXAMPLE 3 **Using the Quotient Rule**

Simplify. Assume that all variables represent positive real numbers.

Solution:

$$\sqrt{\frac{100}{81}} = \frac{10}{9}$$

$$\sqrt{\frac{11}{25}} = \frac{\sqrt{11}}{5}$$

$$\sqrt[3]{\frac{18}{125}} = \frac{\sqrt[3]{18}}{\sqrt[3]{125}} = \frac{\sqrt[3]{18}}{5}$$

$$\sqrt{\frac{y^8}{16}} = \frac{\sqrt{y^8}}{\sqrt{16}} = \frac{y^4}{4}$$

$$-\sqrt[3]{\frac{x^2}{r^{12}}} = -\frac{\sqrt[3]{x^2}}{\sqrt[3]{r^{12}}} = -\frac{\sqrt[3]{x^2}}{r^4}$$

CLASSROOM EXAMPLE 4 **Simplifying Roots of Numbers**

Simplify.

Solution:

$\sqrt{32} = \sqrt{16 \cdot 2} = \sqrt{16} \cdot \sqrt{2} = 4\sqrt{2}$

$\sqrt{300} = \sqrt{100 \cdot 3} = \sqrt{100} \cdot \sqrt{3} = 10\sqrt{3}$

$\sqrt{35}$ Cannot be simplified further.

$\sqrt[3]{54} = \sqrt[3]{27 \cdot 2} = \sqrt[3]{27} \cdot \sqrt[3]{2} = 3\sqrt[3]{2}$

$\sqrt[4]{243} = \sqrt[4]{3^4 \cdot 3} = 3\sqrt[4]{3}$

Simplify radicals.

Conditions for a Simplified Radical

1. The radicand has no factor raised to a power greater than or equal to the index.
2. The radicand has no fractions.
3. No denominator contains a radical.
4. Exponents in the radicand and the index of the radical have greatest common factor 1.

CAUTION ***Be careful with which factors belong outside the radical sign and which belong inside.***

CLASSROOM EXAMPLE 5 **Simplifying Radicals Involving Variables**

Simplify. Assume that all variables represent positive real numbers.

Solution:

$\sqrt{25p^7} = \sqrt{5^2 \cdot (p^3)^2 \cdot p} = 5p^3\sqrt{p}$

$\sqrt{72y^3x} = \sqrt{36 \cdot 2 \cdot y^2 \cdot y \cdot x} = 6y\sqrt{2yx}$

$\sqrt[3]{-27y^7x^5z^6} = \sqrt[3]{-3^3 \cdot y^6 \cdot y \cdot x^3 \cdot x^2 \cdot z^6} = -3y^2xz^2\sqrt[3]{yx^2}$

$-\sqrt[4]{32a^5b^7} = -\sqrt[4]{2^4 \cdot 2 \cdot a^4 \cdot a \cdot b^4 \cdot b^3} = -2ab\sqrt[4]{2ab^3}$

CLASSROOM EXAMPLE 6 **Simplifying Radicals by Using Smaller Indexes**

Simplify. Assume that all variables represent positive real numbers.

Solution:

$\sqrt[12]{2^3} = \left(2^3\right)^{1/12} = 2^{1/4} = \sqrt[4]{2}$

$\sqrt[6]{t^2} = \left(t^2\right)^{1/6} = \left(t\right)^{1/3} = \sqrt[3]{t}$

Simplify radicals.

$\sqrt[kn]{a^{km}}$

If m is an integer, n and k are natural numbers, and all indicated roots exist, then

$$\sqrt[kn]{a^{km}} = \sqrt[n]{a^m}.$$

CLASSROOM EXAMPLE 7 **Multiplying Radicals with Different Indexes**

Simplify.

Solution:

$\sqrt{5} \cdot \sqrt[3]{4}$ The indexes, 2 and 3, have a least common index of 6, use rational exponents to write each radical as a sixth root.

$= 5^{1/2} = 5^{3/6} = \sqrt[6]{5^3} = \sqrt[6]{125}$

$= 4^{1/3} = 4^{2/6} = \sqrt[6]{4^2} = \sqrt[6]{16}$

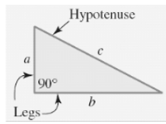
$\sqrt{5} \cdot \sqrt[3]{4} = \sqrt[6]{125} \cdot \sqrt[6]{16} = \sqrt[6]{2000}$

Use the Pythagorean theorem.

Pythagorean Theorem

If c is the length of the longest side of a right triangle and a and b are lengths of the shorter sides, then

$c^2 = a^2 + b^2$.

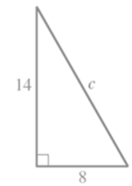


The longest side is the **hypotenuse**, and the two shorter sides are the **legs**, of the triangle. The hypotenuse is the side opposite the right angle.

CLASSROOM EXAMPLE 8 **Using the Pythagorean Theorem**

Find the length of the unknown side of the triangle.

Solution:



$c^2 = a^2 + b^2$
$c^2 = 14^2 + 8^2$
$c^2 = 196 + 64$
$c^2 = 260$

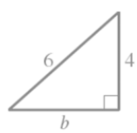
$$c = \sqrt{260}$$
$$c = \sqrt{4 \cdot 65}$$
$$c = \sqrt{4} \cdot \sqrt{65}$$
$$c = 2\sqrt{65}$$

The length of the hypotenuse is $2\sqrt{65}$.

CLASSROOM EXAMPLE 8 **Using the Pythagorean Theorem (cont'd)**

Find the length of the unknown side of the triangle.

Solution:



$c^2 = a^2 + b^2$
$6^2 = 4^2 + b^2$
$36 = 16 + b^2$
$20 = b^2$

$$\sqrt{20} = b$$
$$\sqrt{4 \cdot 5} = b$$
$$\sqrt{4} \cdot \sqrt{5} = b$$
$$2\sqrt{5} = b$$

The length of the leg is $2\sqrt{5}$.

Use the distance formula.

Distance Formula

The distance between points (x_1, y_1) and (x_2, y_2) is

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

CLASSROOM EXAMPLE 9 **Using the Distance Formula**

Find the distance between each pair of points.
(2, –1) and (5, 3)
Solution:
Designate which points are (x_1, y_1) and (x_2, y_2).
$(x_1, y_1) = (2, -1)$ and $(x_2, y_2) = (5, 3)$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$d = \sqrt{(5-2)^2 + (3-(-1))^2}$$

Start with the *x*-value and *y*-value of the *same* point.

$$d = \sqrt{(3)^2 + (4)^2}$$
$$d = \sqrt{9+16}$$
$$d = \sqrt{25} = 5$$

CLASSROOM EXAMPLE 9 **Using the Distance Formula (cont'd)**

Find the distance between each pairs of points.
(–3, 2) and (0, –4)
Solution:
Designate which points are (x_1, y_1) and (x_2, y_2).
$(x_1, y_1) = (-3, 2)$ and $(x_2, y_2) = (0, -4)$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_2)^2}$$
$$d = \sqrt{(0-(-3))^2 + (-4-2)^2}$$
$$d = \sqrt{(3)^2 + (-6)^2}$$
$$d = \sqrt{9+36}$$
$$d = \sqrt{45} = 3\sqrt{5}$$