9.4 Formulas and Further Applications

Objectives

1. Solve formulas for variables involving squares and square roots.
2. Solve applied problems using the Pythagorean theorem.
3. Solve applied problems using area formulas.
4. Solve applied problems using quadratic functions as models.

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Objective 1

Solve formulas for variables involving squares and square roots.

CLASSROOM EXAMPLE 1 **Solving for Variables Involving Squares or Square Roots**

Solve the formula for the given variable. Keep ± in the answer.

Solve $A = \pi r^2$ for *r*.

Solution:

$$\frac{A}{\pi} = r^2$$

$$r = \pm\sqrt{\frac{A}{\pi}}$$

$$r = \pm\frac{\sqrt{A}}{\sqrt{\pi}} \cdot \frac{\sqrt{\pi}}{\sqrt{\pi}}$$

$$r = \pm\frac{\sqrt{A\pi}}{\pi}$$

CLASSROOM EXAMPLE 1 **Solving for Variables Involving Squares or Square Roots (cont'd)**

Solve the formula for the given variable.

Solve $s = 30\sqrt{\frac{a}{p}}$ for *a*.

Solution:

$$s^2 = 900 \cdot \frac{a}{p}$$ **Square both sides.**

$$ps^2 = 900a$$ **Multiply by *p*.**

$$\frac{ps^2}{900} = a$$ **Divide by 900.**

CLASSROOM EXAMPLE 2 **Solving for a Variable That Appears in First- and Second-Degree Terms**

Solve $2t^2 - 5t + k = 0$ for *t*.

Solution:

Use *a* = 2, *b* = –5, and *c* = *k* in the quadratic formula.

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$t = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(2)k}}{2(2)}$$

$$t = \frac{5 \pm \sqrt{25 - 8k}}{4}$$

The solutions are $t = \frac{5 + \sqrt{25 - 8k}}{4}$ and $t = \frac{5 - \sqrt{25 - 8k}}{4}$.

Objective 2

Solve applied problems using the Pythagorean theorem.

CLASSROOM EXAMPLE 3

Using the Pythagorean Theorem

A ladder is leaning against a house. The distance from the bottom of the ladder to the house is 5 ft. The distance from the top of ladder to the ground is 1 ft less than the length of the ladder. How long is ladder?

Solution:

Step 1 **Read** the problem carefully.

Step 2 **Assign the variable.**

Let x = the length of the ladder. Then, $x - 1$ = the distance from the top of the ladder to the ground.

CLASSROOM EXAMPLE 3

Using the Pythagorean Theorem (cont'd)

Step 3 **Write an equation.**

The wall of the house is perpendicular to the ground, so this is a right triangle. Use the Pythagorean formula.

$$a^2 + b^2 = c^2$$

$$5^2 + (x-1)^2 = x^2$$

Step 4 **Solve.**

$$25 + x^2 - 2x + 1 = x^2$$

$$26 = 2x$$

$$13 = x$$

CLASSROOM EXAMPLE 3

Using the Pythagorean Theorem (cont'd)

Step 5 **State the answer.**

The length of the ladder is 13 feet and the distance of the top of the ladder to the ground is 12 feet.

Step 6 **Check.**

$5^2 + 12^2 = 13^2$ and 12 is one less than 13, as required.

Objective 3

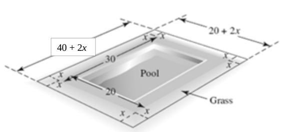
Solve applied problems using area formulas.

CLASSROOM EXAMPLE 4

Solving an Area Problem

Suppose the pool is 20 ft by 40 ft. The homeowner wants to plant a strip of grass around the edge of the pool. There is enough seed to cover 700 ft^2. How wide should the grass strip be?

Solution:



Step 1 **Read** the problem carefully.

Step 2 **Assign the variable.**

Let x = the width of the grass strip

CLASSROOM EXAMPLE 4

Solving an Area Problem (cont'd)

Step 3 **Write an equation.** The width of the larger rectangle is 20 + 20x, and the length is 40 + 2x.

Area of the rectangle – area of pool = area of grass

$$(20+2x)(40+2x) - 20(40) = 700$$

Step 4 **Solve.**

$$800 + 120x + 4x^2 - 800 = 700$$

$$4x^2 + 120x - 700 = 0$$

$$x^2 + 30x - 175 = 0$$

$$(x+35)(x-5) = 0$$

$$x + 35 = 0 \quad \text{or} \quad x - 5 = 0$$

$$x = -35 \quad \text{or} \quad x = 5$$

CLASSROOM EXAMPLE 4 — **Solving an Area Problem (cont'd)**

Step 5 **State the answer.**

The width cannot be –35, so the grass strip should be 5 feet wide.

Step 6 **Check.**

If $x = 5$, then the area of the large rectangle is (40 + 2 • 5) = 50 • 30 = 1500 ft^2.

The area of the pool is 40 • 20 = 800ft^2.

So, the area of the grass strip is 1500 – 800 = 700 ft^2, as required. The answer is correct.

CLASSROOM EXAMPLE 5 — **Solving an Applied Problem Using a Quadratic Function**

A ball is projected upward from the ground. Its distance in feet from the ground at t seconds is $s(t) = -16t^2 + 64t$. At what time will the ball be 32 feet from the ground?

Solution:

$$s(t) = -16t^2 + 64t$$

$$32 = -16t^2 + 64t$$

$$16t^2 - 64t + 32 = 0$$

$$t^2 - 4t + 2 = 0$$

CLASSROOM EXAMPLE 5 — **Solving an Applied Problem Using a Quadratic Function (cont'd)**

Use $a = 1$, $b = -4$, and $c = 2$ in the quadratic formula.

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$t = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(2)}}{2(1)} = \frac{4 \pm \sqrt{16-8}}{2}$$

$$= \frac{4 \pm \sqrt{8}}{2} = \frac{4 \pm 2\sqrt{2}}{2} = 2 \pm \sqrt{2}$$

The solutions are $t = 2 + \sqrt{2} \approx 3.4$ or $2 - \sqrt{2} \approx 0.6$.

The ball will be at a height of 32 ft at about 0.6 seconds and 3.4 seconds.

CLASSROOM EXAMPLE 6 — **Using a Quadratic Function to Model the CPI**

The Consumer Price Index (CPI) is used to measure trends in prices for a "basket" of goods purchased by typical American families. This index uses a base year of 1967, which means that the index number for 1967 is 100. The quadratic function defined by

$$f(x) = -0.065x^2 + 14.8x + 249$$

approximates the CPI for the years 1980-2005, where x is the number of years that have elapsed since 1980.
(*Source:* Bureau of Labor Statistics.)

Use the model to approximate the CPI for 2000, to the nearest whole number.

In what year did the CPI reach 450? (Round down for the year.)

CLASSROOM EXAMPLE 6 — **Using a Quadratic Function to Model the CPI (cont'd)**

Use the model to approximate the CPI for 2000, to the nearest whole number.

$$f(x) = -0.065x^2 + 14.8x + 249$$

Solution:

For 2000, x = 2000 − 1980, so find $f(20)$.

$$f(20) = -0.065(20)^2 + 14.8(20) + 249$$

$$f(20) = -26 + 296 + 249$$

$$f(20) \approx 519$$

The CPI for 2000 was about 519.

CLASSROOM EXAMPLE 6 — **Using a Quadratic Function to Model the CPI (cont'd)**

In what year did the CPI reach 450? (Round down for the year.)

$$f(x) = -0.065x^2 + 14.8x + 249$$

$$450 = -0.065x^2 + 14.8x + 249$$

$$0 = -0.065x^2 + 14.8x - 201$$

$$x = \frac{-14.8 \pm \sqrt{14.8^2 - 4(-0.065)(-201)}}{2(-0.065)}$$

$$x \approx 14.5 \text{ or } x \approx 213.2$$

The CPI first reached 450 in 1980 + 14yr = 1994. (The second solution is rejected as 1980 + 213 = 2192, which is far beyond period covered by the model.