9.5 Graphs of Quadratic Functions

Objectives

1. Graph a quadratic function.
2. Graph parabolas with horizontal and vertical shifts.
3. Use the coefficient of x^2 to predict the shape and direction in which a parabola opens.
4. Find a quadratic function to model data.

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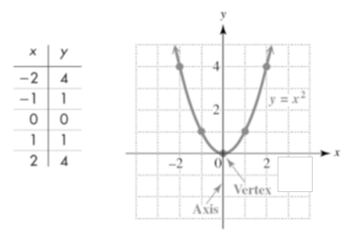
Graph a quadratic function.

The graph shown below is a graph of the simplest ***quadratic function,*** defined by $y = x^2$.

This graph is called a **parabola**.



The point (0, 0), the lowest point on the curve, is the **vertex.**

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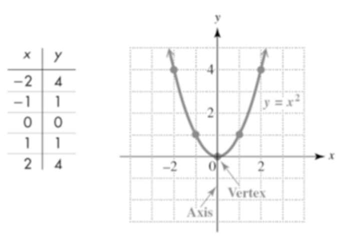
Graph a quadratic function.

The vertical line through the vertex is the **axis** of the parabola, here $x = 0$.

A parabola is **symmetric about its axis**.



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Graph parabolas with horizontal and vertical shifts.

Quadratic Function

A function that can be written in the form

$$f(x) = ax^2 + bx + c$$

for real numbers *a, b,* and *c*, with $a \neq 0$, is a **quadratic function.**

The graph of any quadratic function is a parabola with a vertical axis.

We use the variable ***y*** and function notation ***f (x)*** interchangeably. Although we use the letter ***f*** most often to name quadratic functions, other letters can be used. We use the capital letter ***F*** to distinguish between different parabolas graphed on the same coordinate axes.

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Graph parabolas with horizontal and vertical shifts.

Parabolas do not need to have their vertices at the origin.

The graph of

$$F(x) = x^2 + k$$

is shifted, or translated *k* units vertically compared to $f(x) = x^2$.

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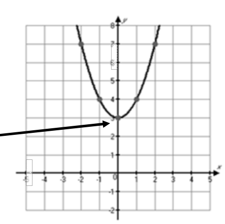
CLASSROOM EXAMPLE 1 **Graphing a Parabola (Vertical Shift)**

Graph $f(x) = x^2 + 3$. Give the vertex, domain, and range.
Solution:

The graph has the same shape as $f(x) = x^2$, but shifted up 3 units.

Make a table of points.

x	$x^2 + 3$
–2	7
–1	4
0	3
1	4
2	7



vertex (0, 3)

domain: $(-\infty, \infty)$

range: $[3, \infty)$

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Graph parabolas with horizontal and vertical shifts.

Vertical Shift

The graph of $F(x) = x^2 + k$ is a parabola.

- The graph has the same shape as the graph of $f(x) = x^2$.
- The parabola is shifted k units up if $k > 0$, and $|k|$ units down if $k < 0$.
- The vertex is $(0, k)$.

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CLASSROOM EXAMPLE 2 Graphing a Parabola (Horizontal Shift)

Graph $f(x) = (x + 2)^2$. Give the vertex, axis, domain, and range.

Solution:

The graph has the same shape as $f(x) = x^2$, but shifted 2 units to the left.

Make a table of points.

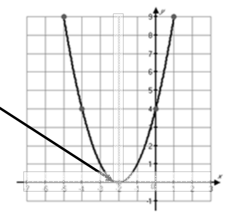
x	$(x + 2)^2$
–5	9
–4	4
–2	0
0	4
1	9

vertex (–2, 0)

axis $x = -2$

domain: $(-\infty, \infty)$

range: $[0, \infty)$



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Graph parabolas with horizontal and vertical shifts.

Horizontal Shift

The graph of $F(x) = (x - h)^2$ is a parabola.

- The graph has the same shape as the graph of $f(x) = x^2$.
- The parabola is shifted h units to the right if $h > 0$, and $|h|$ units to the left if $h < 0$.
- The vertex is $(h, 0)$.

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CLASSROOM EXAMPLE 3 Graphing a Parabola (Horizontal and Vertical Shifts)

Graph $f(x) = (x - 2)^2 + 1$. Give the vertex, axis, domain, and range.

Solution:

The graph has the same shape as $f(x) = x^2$, but shifted 2 units to the right and 3 unit up.

Make a table of points.

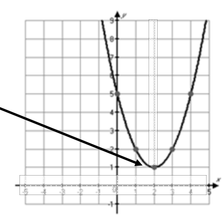
x	$f(x)$
0	5
1	2
2	1
3	2
4	5

vertex (2, 1)

axis $x = 2$

domain: $(-\infty, \infty)$

range: $[1, \infty)$



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Graph parabolas with horizontal and vertical shifts.

Vertex and Axis of Parabola

The graph of $F(x) = (x - h)^2 + k$ is a parabola.

- The graph has the same shape as the graph of $f(x) = x^2$.
- The vertex of the parabola is (h, k).
- The axis is the vertical line $x = h$.

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Objective 3

Use the coefficient of x^2 to predict the shape and direction in which a parabola opens.

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CLASSROOM EXAMPLE 4

Graphing a Parabola That Opens Down

Graph $f(x) = -2x^2 - 3$. Give the vertex, axis, domain, and range.

Solution:

The coefficient (–2) affects the shape of the graph; the 2 makes the parabola narrower.

The negative sign makes the parabola open down.

The graph is shifted down 3 units.

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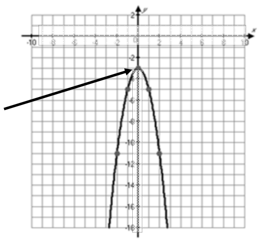
CLASSROOM EXAMPLE 4

Graphing a Parabola That Opens Down (cont'd)

Graph $f(x) = -2x^2 - 3$.

x	$f(x)$
–2	–11
–1	–5
0	–3
1	–
2	–11

vertex (0, –3)



axis $x = 0$ domain: $(-\infty, \infty)$ range: $(-\infty, -3]$

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Use the coefficient of x^2 to predict the shape and direction in which a parabola opens.

General Principles of $F(x) = a(x - h)^2 + k$ ($a \neq 0$)

1. The graph of the quadratic function defined by

$$F(x) = a(x - h)^2 + k,\ a \neq 0,$$

is a parabola with vertex (h, k) and the vertical line $x = h$ as axis.

2. The graph opens up if a is positive and down if a is negative.

3. The graph is wider than that of $f(x) = x^2$ if $0 < |a| < 1$.
The graph is narrower than that of $f(x) = x^2$ if $|a| > 1$.

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CLASSROOM EXAMPLE 5

Using the General Characteristics to Graph a Parabola

Graph $f(x) = \frac{1}{2}(x - 2)^2 + 1.$

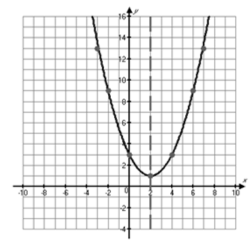
Solution:

Parabola opens up.
Narrower than $f(x) = x^2$
Vertex: (2, 1)

axis $x = 2$

domain: $(-\infty, \infty)$

range: $[1, \infty)$



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