How to find the area between two curves within $a \le x \le b$

Step 1) Find the points of intersections between 2 curves.

Step 2) Graph both functions

Step 3) Within $a \le x \le b$ and intersections points see how many different areas you have and which one is at the op and which one is a the bottom?

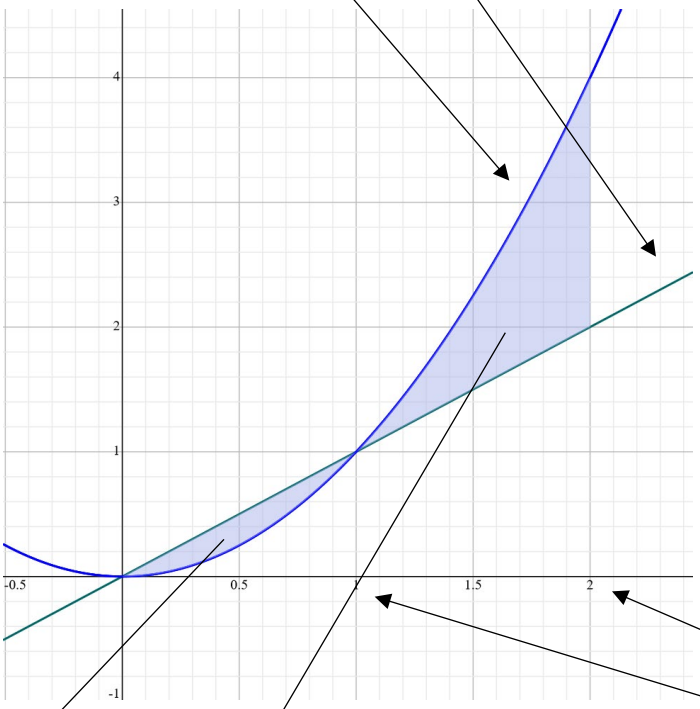
Step 4) Construct the integral for the number of observable areas knowing that for each area we have to express the **function of top area *minus* the function of the bottom area** with appropriate limits

Find the area between $f(x) = x^2$ and $g(x) = x$ bounded by $0 \le x \le 2$

Step 1) Find the points of intersections between 2 curves.

$f(x) = x^2 = g(x) = x \quad \rightarrow \quad x^2 = x \quad \rightarrow \quad x^2 - x = 0 \qquad x = 1, \quad x = 0$

Step 2) Graph both functions



Step 3) Within $0 \le x \le 1 \quad g(x) = x$ is at the top and $f(x) = x^2$ at the bottom and within $1 \le x \le 2$

$f(x) = x^2$ is at the top and $g(x) = x$ at the bottom,

Step 4) $\int_0^1 [g(x) - f(x)]\, dx \; + \int_1^2 [f(x) - g(x)] = \frac{1}{6} + \frac{5}{6} = 1$

 Name_____

Area Between Curves

Date_____ Period____

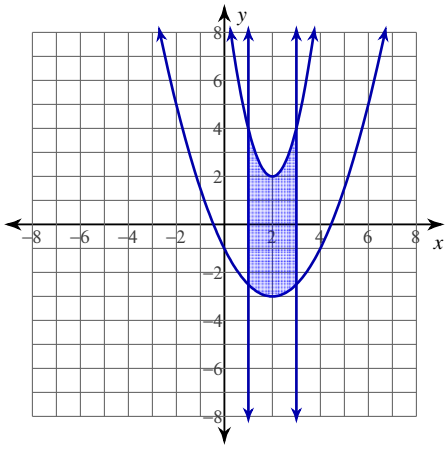
For each problem, find the area of the region enclosed by the curves.

1) $y = 2x^2 - 8x + 10$

$y = \frac{x^2}{2} - 2x - 1$

$x = 1$

$x = 3$

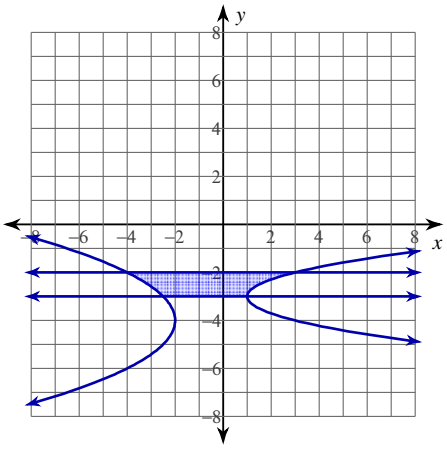


2) $x = 2y^2 + 12y + 19$

$x = -\frac{y^2}{2} - 4y - 10$

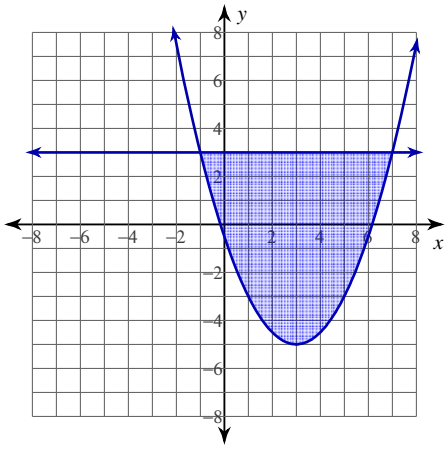
$y = -3$

$y = -2$



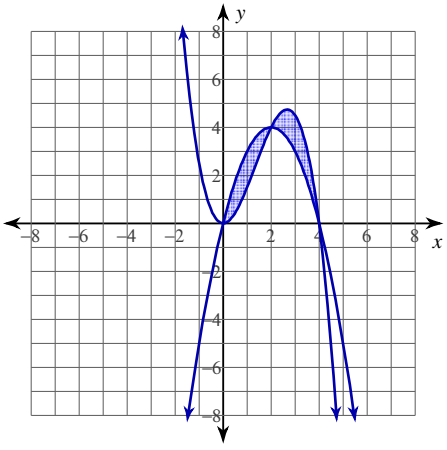
3) $y = \frac{x^2}{2} - 3x - \frac{1}{2}$

$y = 3$



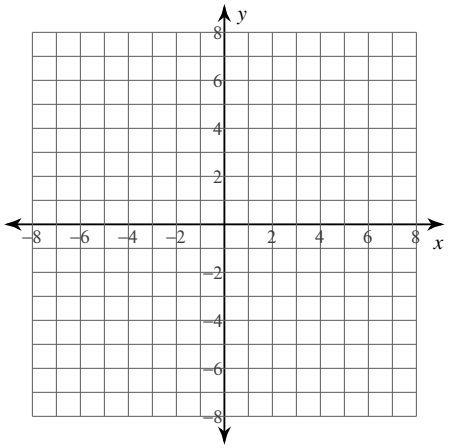
4) $y = -\frac{x^3}{2} + 2x^2$

$y = -x^2 + 4x$

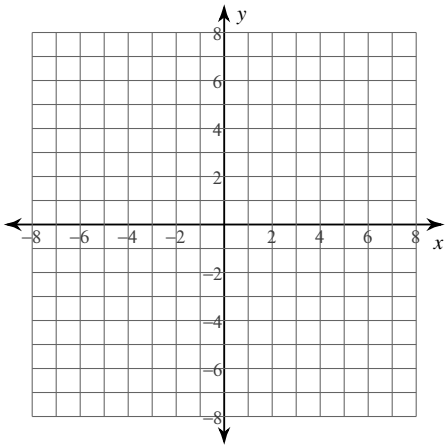


For each problem, find the area of the region enclosed by the curves. You may use the provided graph to sketch the curves and shade the enclosed region.

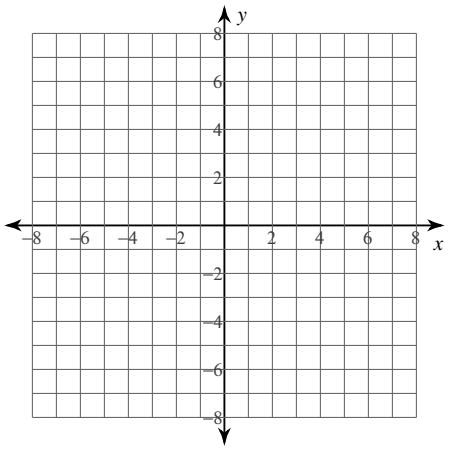
5) $y = -2x^2 - 1$
$y = -x + 3$
$x = 0$
$x = 1$



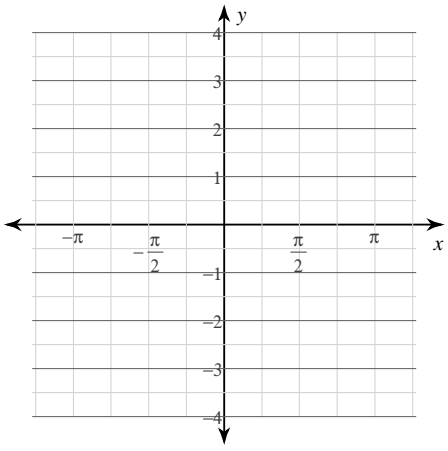
6) $y = 2\sqrt[3]{x^2}$
$y = x$



7) $y = -x^3 + 6x$
$y = -x^2$



8) $y = -2 \cdot \sec^2 x$
$y = 2\cos x$
$x = 0$
$x = \dfrac{\pi}{4}$



 Name_____

Area Between Curves

Date_____ Period____

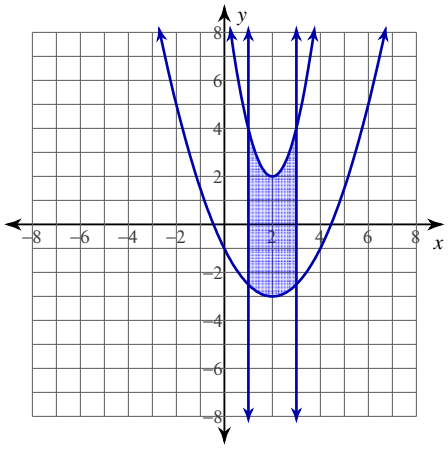
For each problem, find the area of the region enclosed by the curves.

1) $y = 2x^2 - 8x + 10$

$y = \frac{x^2}{2} - 2x - 1$

$x = 1$

$x = 3$



$\int_1^3 \left(2x^2 - 8x + 10 - \left(\frac{x^2}{2} - 2x - 1\right)\right) dx$

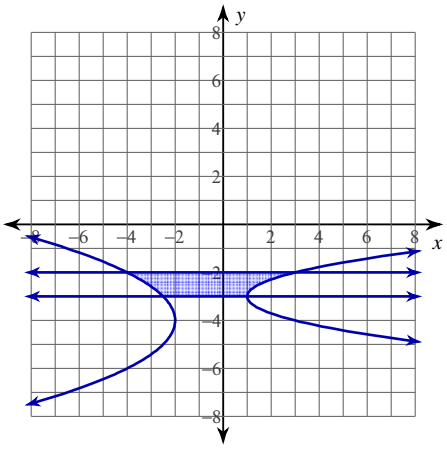
$= 11$

2) $x = 2y^2 + 12y + 19$

$x = -\frac{y^2}{2} - 4y - 10$

$y = -3$

$y = -2$

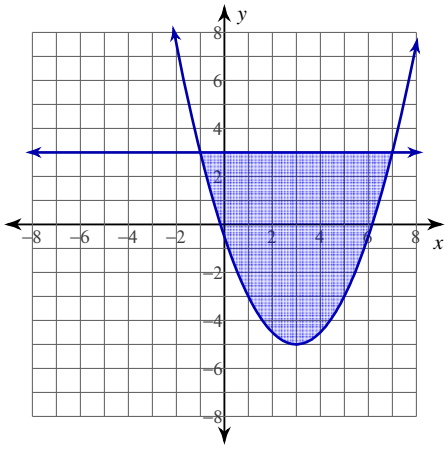


$\int_{-3}^{-2} \left(2y^2 + 12y + 19 - \left(-\frac{y^2}{2} - 4y - 10\right)\right) dy$

$= \frac{29}{6} \approx 4.833$

3) $y = \frac{x^2}{2} - 3x - \frac{1}{2}$

$y = 3$

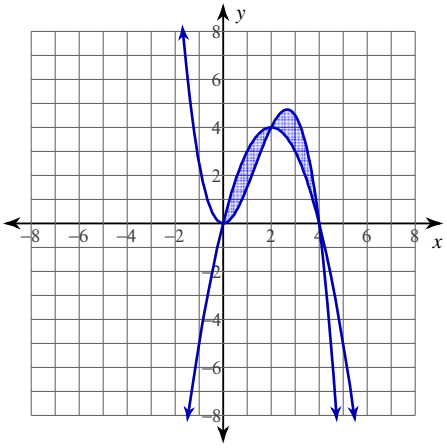


$\int_{-1}^{7} \left(3 - \left(\frac{x^2}{2} - 3x - \frac{1}{2}\right)\right) dx$

$= \frac{128}{3} \approx 42.667$

4) $y = -\frac{x^3}{2} + 2x^2$

$y = -x^2 + 4x$



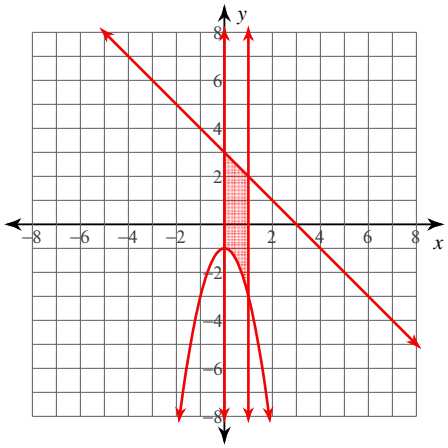
$\int_0^2 \left(-x^2 + 4x - \left(-\frac{x^3}{2} + 2x^2\right)\right) dx +$

$\int_2^4 \left(-\frac{x^3}{2} + 2x^2 - \left(-x^2 + 4x\right)\right) dx$

$= 4$

For each problem, find the area of the region enclosed by the curves. You may use the provided graph to sketch the curves and shade the enclosed region.

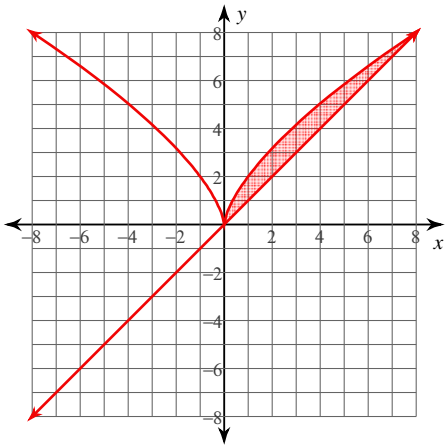
5) $y = -2x^2 - 1$
$y = -x + 3$
$x = 0$
$x = 1$



$\int_0^1 (-x + 3 - (-2x^2 - 1))\, dx$

$= \frac{25}{6} \approx 4.167$

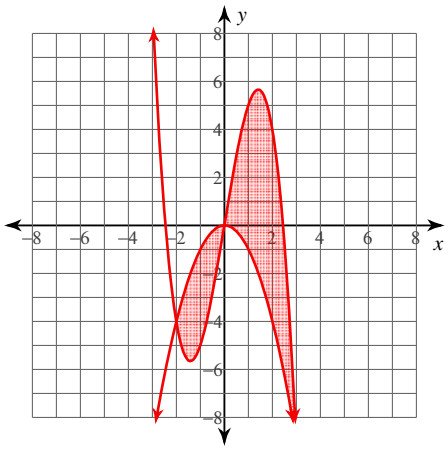
6) $y = 2\sqrt[3]{x^2}$
$y = x$



$\int_0^8 (2\sqrt[3]{x^2} - x)\, dx$

$= \frac{32}{5} = 6.4$

7) $y = -x^3 + 6x$
$y = -x^2$

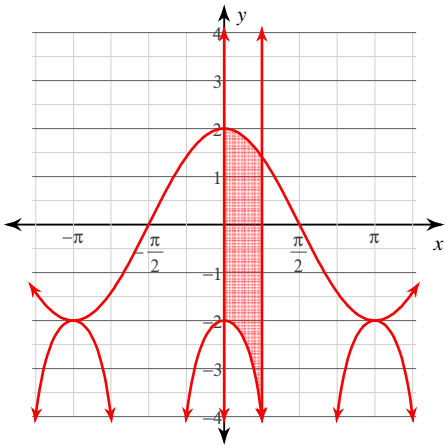


$\int_{-2}^0 (-x^2 - (-x^3 + 6x))\, dx +$

$\int_0^3 (-x^3 + 6x + x^2)\, dx$

$= \frac{253}{12} \approx 21.083$

8) $y = -2 \cdot \sec^2 x$
$y = 2\cos x$
$x = 0$
$x = \frac{\pi}{4}$



$\int_0^{\frac{\pi}{4}} (2\cos x + 2 \cdot \sec^2 x)\, dx$

$= 2 + \sqrt{2} \approx 3.414$

Area Between Two Curves

SUGGESTED REFERENCE MATERIAL:

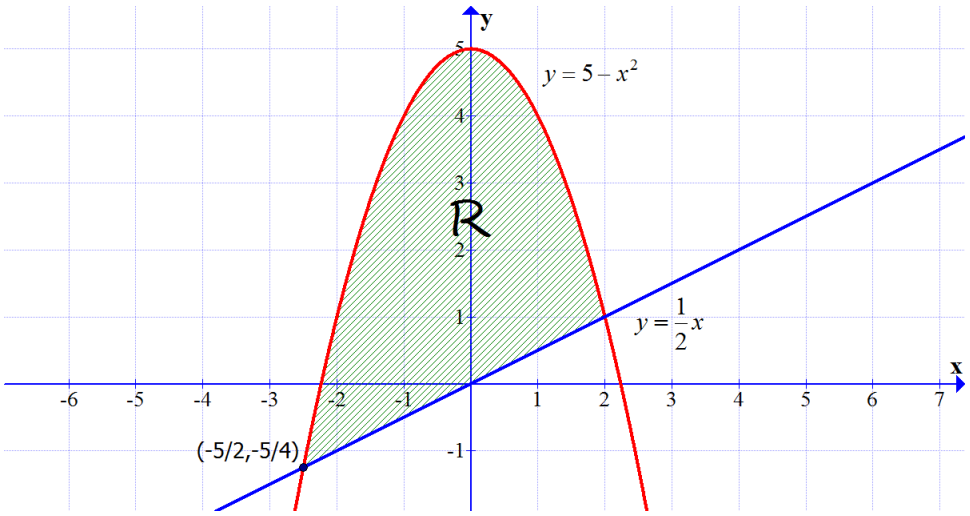
As you work through the problems listed below, you should reference Chapter 6.1 of the recommended textbook (or the equivalent chapter in your alternative textbook/online resource) and your lecture notes.

EXPECTED SKILLS:

- Be able to find the area between the graphs of two functions over an interval of interest.
- Know how to find the area enclosed by two graphs which intersect.

PRACTICE PROBLEMS:

1. Let R be the shaded region shown below.

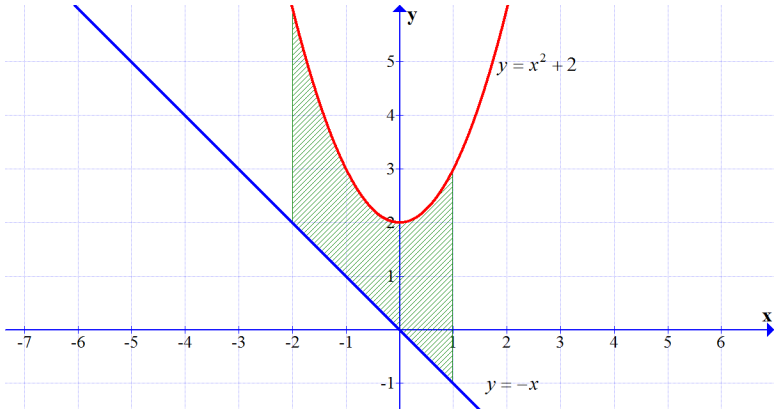


   (a) Set up but do not evaluate an integral (or integrals) in terms of x that represent(s) the area of R.

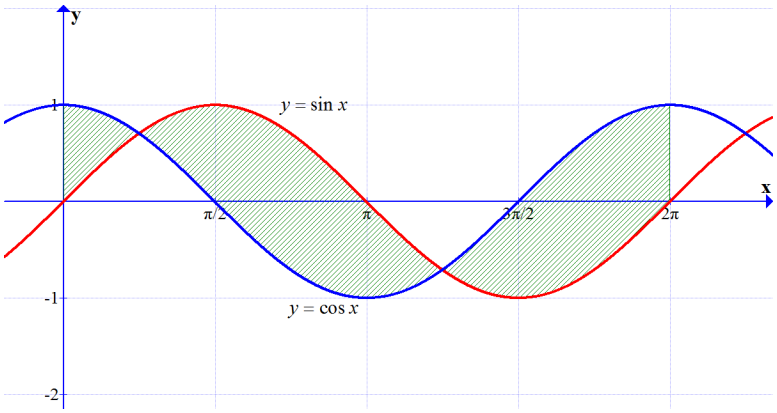
   (b) Set up but do not evaluate an integral (or integrals) in terms of y that represent(s) the area of R.

For problems 2-4, compute the area of the shaded region.

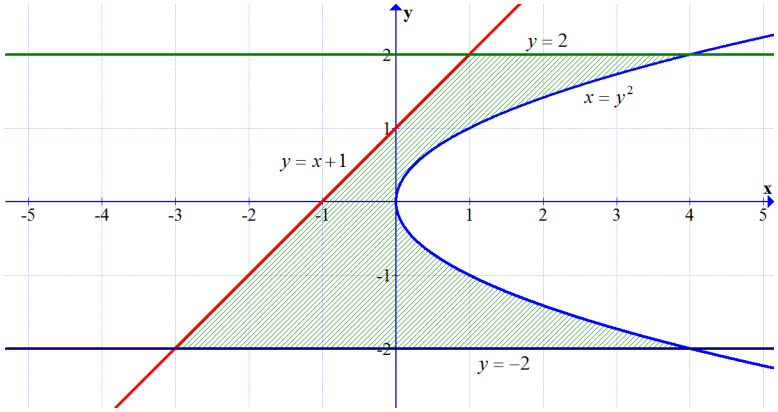
2.



3.



4.



For problems 5-13, compute the area of the region which is enclosed by the given curves.

5. $y = 4x$, $y = 6x^2$
6. $y = 2x^2$, $y = x^2 + 2$
7. $y = x^{2/3}$, $y = x^4$, in the first quadrant
8. $y = \dfrac{1}{x}$, $y = \dfrac{1}{x^2}$, $x = 4$
9. $y = \sin x$, $y = 2 - \sin x$, $\dfrac{\pi}{2} \leq x \leq \dfrac{5\pi}{2}$
10. $y = e^{5x}$, $y = e^{8x}$, $x = 1$
11. $x = 4 - y^2$, $x = y^2 - 4$
12. $y = x^4$, $y = |x|$
13. $y = x^2$, $y = \dfrac{2}{x^2 + 1}$