Business Functions

In Business, the following functions are important.
Revenue function = (price per unit) . (quantity of units)
Symbols: $\boxed{R = p \, . \, x}$
Cost function = (average cost per unit) . (quantity of units)
Symbols: $\boxed{C = \overline{C} \, . \, x}$
Profit function = revenue − cost
Symbols: $\boxed{P = R - C}$
Sometimes in a problem some of these functions are given.
Note: Do not confuse p and P.
The price per unit p is also called the demand function p.

Marginal Functions:
The derivative of a function is called marginal function.

The derivative of the revenue function $R(x)$ is called marginal revenue with notation: $\boxed{R'(x) = \dfrac{dR}{dx}}$

The derivative of the cost function $C(x)$ is called marginal cost with notation: $\boxed{C'(x) = \dfrac{dC}{dx}}$

The derivative of the profit function $P(x)$ is called marginal profit with notation: $\boxed{P'(x) = \dfrac{dP}{dx}}$

Example 1: Given the price in dollar per unit $p = -3x^2 + 600x$, find:

(a) the marginal revenue at $x = 300$ units. Interpret the result.

revenue function: $R(x) = p \, . \, x = (-3x^2 + 600x) \, . \, x = -3x^3 + 600x^2$

marginal revenue: $R'(x) = \dfrac{dR}{dx} = -9x^2 + 1200x$

marginal revenue at $x = 300 \implies R'(300) = \left.\dfrac{dR}{dx}\right|_{x=300} = -9(300)^2 + 1200(300) = -450\,000$

Interpretation: If production increases from 300 to 301 units, the revenue decreases by 450 000 dollars.

(b) the marginal revenue at $x = 100$ units. Interpret the result.

revenue function: $R(x) = p \, . \, x = (-3x^2 + 600x) \, . \, x = -3x^3 + 600x^2$

marginal revenue: $R'(x) = \dfrac{dR}{dx} = -9x^2 + 1200x$

marginal revenue at $x = 100 \implies R'(100) = \left.\dfrac{dR}{dx}\right|_{x=100} = -9(100)^2 + 1200(100) = 30\,000$

Interpretation: If production increases from 100 to 101 units, the revenue increases by 30 000 dollars.

4.2.8 A company manufactures and sells scooters. The company finds that the fixed costs for producing its product are $211008 per month and the variable costs are $47 per scooter. The company sells its scooters for $215.

Problem 1: Profit function

Cost Function: $C(x) = 47x + 211008$

Revenue Function: $R(x) = 215x$

Profit function:

$\boxed{P(x) = R(x) - C(x) = 168x - 211008}$

Break even point:

Command: root[R(x)] **Answer:** (1256, 270040)

Problem 2: Break even quantity

$\boxed{1256}$ scooters

Problem 3: Break even Revenue

$\boxed{\$270040}$

Problem 4: 4.2.40, Part A

The demand for a company's product can be modeled by the function $p = 500 - 0.03x$. Find the revenue function.

$\boxed{R(x) = x * p = 500x - 0.03x^2}$

4.2.48 The total daily cost to produce x units of a company's product can be modeled by $C(x) = -0.001x^3 - 0.02x^2 + 27x + 3800$ where C(x) is measured in dollars. The demand for the product can be modeled by $p = 62 - 0.03x$.

Problem 5: 4.2.48. Part A

Find the profit function.

Revenue function: $R(x) = x * p = 62x - 0.03x^2$

$P(x) = R(x) - C(x) = (62x - 0.03x^2) - (-0.001x^3 - 0.02x^2 + 27x + 3800)$

$\boxed{P(x) = 0.001x^3 - 0.01x^2 + 35x - 3800}$

Problem 6: 4.2.48 Find the break-even quantity

Command: Root [P(x)] Answer: (90.0348, 0)

break-even quantity

$\boxed{90}$

Example 2: Given the average cost in dollar per unit $\overline{C} = 357x + 1800$, find:

the marginal cost at $x = 50$ units. Interpret the result.

cost function: $C(x) = \overline{C} \,.\, x = (357x + 1800)\,.\,x = 357x^2 + 1800x$

marginal cost: $C'(x) = \dfrac{dC}{dx} = 714x + 1800$

marginal cost at $x = 50 \implies C'(50) = \left.\dfrac{dC}{dx}\right|_{x=50} = 714(50) + 1800 = 37\,500$

Interpretation: If production increases from 50 to 51 units, the cost increases by 37 500 dollars.

Example 3: Given the revenue function in dollars $R(x) = -3x^3 + 600x^2$ and the cost function in dollars $C(x) = 357x^2 + 1800x$; find:

(a) the marginal profit at $x = 10$ units. Interpret the result.

profit function = revenue − cost

$P(x) = (-3x^3 + 600x^2) - (357x^2 + 1800x) = -3x^3 + 243x^2 - 1800x$

marginal profit: $P'(x) = \dfrac{dP}{dx} = -9x^2 + 486x - 1800$

marginal profit at $x = 10 \implies P'(10) = \left.\dfrac{dP}{dx}\right|_{x=10} = -9(10)^2 + 486(10) - 1800 = 3300$

Interpretation: If production increases from 10 to 11 units, the profit increases by 3300 dollars.

(b) the marginal profit at $x = 100$ units. Interpret the result.

profit function = revenue − cost

$P(x) = (-3x^3 + 600x^2) - (357x^2 + 1800x) = -3x^3 + 243x^2 - 1800x$

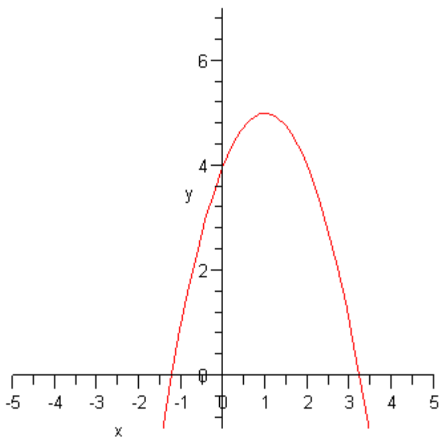
marginal profit: $P'(x) = \dfrac{dP}{dx} = -9x^2 + 486x - 1800$

marginal profit at $x = 100 \implies P'(100) = \left.\dfrac{dP}{dx}\right|_{x=100} = -9(100)^2 + 486(100) - 1800 = -750\,000$

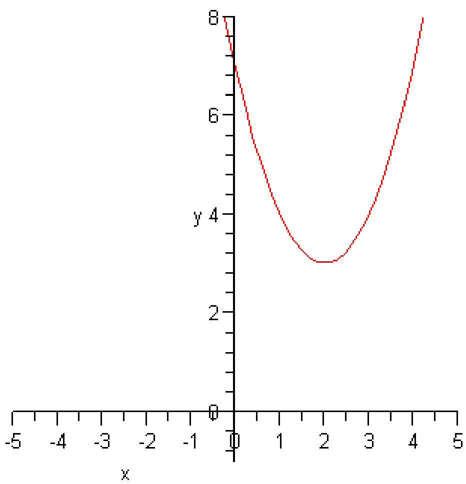
Interpretation: If production increases from 100 to 101 units, the profit decreases by 750 000 dollars.

Optimization (Business Applications)

Maximize Profit



Minimize Cost

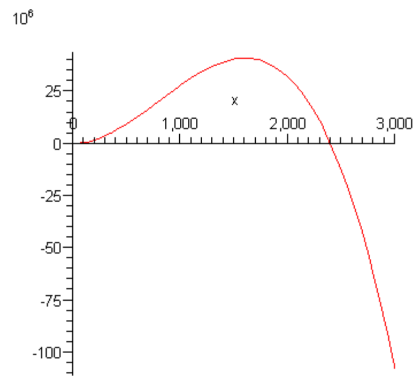


Maximizing Revenue

Example 1

Find the number of units x that produces a maximum

revenue



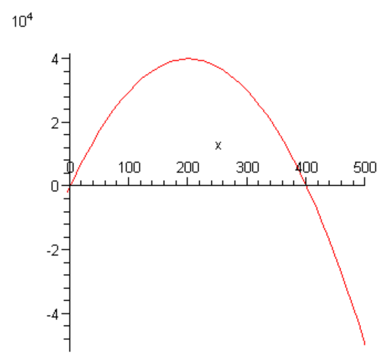
$$R = 48x^2 - 0.02x^3$$
$$R' = 96x - 0.06x^2$$
$$96x - 0.06x^2 = 0$$
$$x(96 - .06x) = 0$$
$$x = 0 \quad or \quad 96 - .06x = 0$$
$$-.06x = -96$$
$$x = 1600$$

Thus, 1600 units will produce a maximum revenue.

Example 2

Let $R = 400 - x^2$ represent the revenue earned by a local outfitter company that sells backpacks where x is the number of backpacks sold in a month.

$$R = 400x - x^2$$
$$R' = 400 - 2x$$



$$400 - 2x = 0$$
$$400 - 2x + 2x = 0 + 2x$$
$$400 = 2x$$
$$\frac{400}{2} = \frac{2x}{2}$$
$$x = 200$$

Thus, selling 200 backpacks in one month would produce a maximum revenue.

Example 3

Given the demand function and cost function below, find the revenue function and then find the value of x that produces the maximum profit. Hint: $R(x) = xp$

$$Demand \;\; Function: \; p = 6000 - 0.4x^2$$
$$Cost \;\; Function: C = 2400x + 5200$$

Solution: First find the revenue function using $R(x) = xp$, then find the profit function by subtracting the cost function from the revenue function. Once you have the profit function, you simply take the derivative of the profit function and set the result equal to zero.

$$Demand \;\; Function$$
$$p = 6000 - 0.4x^2$$
$$C = 2400x + 5200$$
$$R(x) = xp = x(6000 - .4x^2)$$
$$R(x) = 6000x - .x^3$$
$$P(x) = R(x) - C(x)$$
$$P(x) = 6000x - x^3 - (2400x + 5200)$$
$$P(x) = 6000x + .4x^3 - 2400x - 5200$$
$$P(x) = .4x^3 + 3600x - 5200$$
$$P'(x) = .12x^2 + 3600$$
$$-.12x^2 + 3600 = 0$$
$$.12x^2 = 3600$$
$$\frac{.12x^2}{.12} = \frac{3600}{.12}$$
$$x^2 = 3000$$
$$x = \sqrt{3000}$$
$$x = 54.8$$
$$p = 6000 - 54.8^2 = 6000 - 3000 = 3000$$

Example 4

Given the demand function and cost function below, find the revenue function and then find the value of x that produces the maximum profit.

Demand Function

$$p = 50 - .1\sqrt{x} = 50 - .1x^{\frac{1}{2}}$$

$$C = 35x + 500$$

$$R(x) = xp = x(50 - .1x^{\frac{1}{2}})$$

$$R(x) = 50x - ..1x^{\frac{3}{2}}$$

$$P(x) = R(x) - C(x)$$

$$P(x) = 50x - x^{\frac{3}{2}} - (35x + 500)$$

$$P(x) = 50x + .1x^{\frac{3}{2}} - 35x - 500$$

$$P(x) = -.1x^{\frac{3}{2}} + 15x - 500$$

$$P'(x) = -.15x^{\frac{1}{2}} + 15$$

$$-.15\sqrt{x} + 15 = 0$$

$$-.15\sqrt{x} = -15$$

$$\frac{-.15\sqrt{x}}{-.15} = \frac{-15}{.15}$$

$$\sqrt{x} = 100$$

$$x = 10000$$

Optimization problems

How to solve an optimization problem?

1. **Step 1:** Understand the problem and underline what is important (what is known, what is unknown, what we are looking for, *dots*)
2. **Step 2:** Draw a "diagram"; if it is possible.
3. **Step 3:** Assign "symbols" or "**variables**" for all the quantities involved (know or unknown), and label the diagram.
4. **Step 4:** Write the quantity **Q** to be maximized or minimized in terms of some of the previous variables (from Step 3). Example: $\mathbf{Q} = g(x, y, h)$
5. **Step 5:** Rewrite **Q** as a function of **only one variable**. To do this, find the relationships between the variables using the information given in the problem. Then, use these equations to eliminate all but one of the variables in the expression of Q. Thus, we get $Q = f(x)$.
6. **Step 5:** Use the methods of sections 10.1 and 10.2 to find the maximum or the minimum of the quantity $\mathbf{Q} = \mathbf{f}(\mathbf{x})$.
7. **REMARK:** Do not forget to find the endpoints and to check if the maximum or the minimum is at these points if you have more than one critical number in the domain.
8. **Short-cut:** If there is **only one critical** number **a** in the domain, then :

$$\begin{cases} \text{if we have a local maximum at } \mathbf{a} & \Longrightarrow \text{we have a } \textbf{global} \text{ maximum at } \mathbf{a}, \\ \text{if we have a local minimum at } \mathbf{a} & \Longrightarrow \text{we have a } \textbf{global} \text{ minimum at } \mathbf{a}. \end{cases}$$

   In other term, we do not need to check the position of the endpoints.
9. **Reminder:** At the worksheet I gave you in the beginning of the semester (it is the KEY FORMULAS for Chapter 9 posted at the homework assignment web page) of the textbook, you can find all the formulas related to the cost, revenue and profit. You should work

Examples:

Problem 1. The regular air fare between Boston and San Francisco is \$500. An airline using planes with a capacity of 300passengers on this route observes that they fly with an average of 180 passengers. Market research tells the airlines' managers that each \$ 5 fare reduction would attract, on average, 3 more passengers for each flight. How should they set the fare to maximize their revenue? Explain your reasoning to receive credit.

- SOLUTION:

  Let R= the revenue function = quantity × price

  Let n= the number of times the fare is reduced by \$ 5 dollars. Then:

$$\begin{cases} \text{price} & = \$500 - n \cdot (\$5) = 500 - 5n \text{ dollars}, \\ \text{quantity} & = \text{number of passengers} = 180\text{passengers} + n \cdot (3\text{passengers}) = 180 + 3n \text{ passengers}. \end{cases}$$

  Hence $\boxed{R(n) = (180 + 3n)(500 - 5n) = 90\ 000 + 600n - 15n^2}$ to maximize (for $0 \leq n \leq 40$).

  $R'(n) = 600 - 30n = 0 \Rightarrow n = \frac{600}{30} = 20$ is the only critical number.

  $R''(n) = -30 \Rightarrow R''(20) = -30 < 0$. By the second derivative test, R has a local maximum at $n = 20$, which is an absolute maximum since it is the only critical number.

  The best fare to maximize the revenue is then: $\boxed{\$\ 500 - 5(20) = \$400}$, with $\boxed{180 + 3(20) = 240 \text{ passengers}}$ and a revenue of $\boxed{R(20) = \$96,000}$.

Problem 2. A baseball team plays in a stadium that hold $55,000$ spectators. With ticket prices at $\$10$, the average attendance had been $27,000$. A market survey showed that for each $\$0.10$ decrease in the ticket prices, on the average, the attendance will increase by 300. How should ticket prices be set to maximize revenue?

- SOLUTION:

  Let R= the revenue function = quantity × price

  Let n= the number of times the price of the ticket is reduced by \$ 0.10. Then:

  $$\begin{cases} \text{price} & = \$10 - n \cdot (\$0.10) = 10 - 0.10n \text{ dollars,} \\ \text{quantity} & = \text{number of spectators} = 27,000\text{spectators} + n \cdot (300\text{spectators}) = 27,000 + 300n \text{ spectators.} \end{cases}$$

  Hence $\boxed{R(n) = (27,000 + 300n)(10 - 0.10n) = 270,000 + 300n - 30n^2}$ to maximize.

  $R'(n) = 300 - 60n = 0 \Rightarrow n = \frac{300}{60} = 5$ is the only critical number.

  $R''(n) = -60 \Rightarrow R''(5) = -60 < 0$. By the second derivative test, R has a local maximum at $n = 5$, which is an absolute maximum since it is the only critical number.

  The best ticket prices to maximize the revenue is then: $\boxed{\$\ 10 - 0.10(5) = 9.50\ \$}$,

  with $\boxed{27,000 + 300(5) = 28,500}$ spectators and a revenue of $\boxed{\$\ R(5) = 270,750}$.

Problem 3. A Florida Citrus grower estimates that if 60 orange trees are planted; the average yield per tree will be 400 oranges. The average yield will decrease by 4 oranges per tree for each additional tree planted on the same acreage. How many trees should the grower plant to maximize the total yield?

- SOLUTION:

  Let **n**= the number of additional trees. Let **Y**= the total yield = number of trees × the yield per tree. Then:

  $$Y(n) = (60\text{trees} + n \cdot \text{trees})(400\text{oranges} - n \cdot 4\text{oranges}) = (60 + n)(400 - 4n) = 24,000 + 160n - 4n^2$$

  to maximize ! Lets find the critical numbers:

  $Y'(n) = 160 - 8n = 0 \Rightarrow n = \frac{160}{8} = 20$ is the only critical number.

  Moreover, $Y''(n) = -8 \Rightarrow R''(20) = -8 < 0$. By the second derivative test, Y has a local maximum at $n = 20$, which is an absolute maximum since it is the only critical number.

  The grower should plant $\boxed{60+20 = 80}$ trees to maximize the total yield.

Problem 4. A manufacturer of men's shirts determines that her costs will be 500 dollars for overhead plus 9 dollars for each shirt made. Her accountant has estimated that her selling price p should be determined by

$$p = 30 - 0.2\sqrt{x} \qquad \textbf{where } x \textbf{ is the number of shirts sold.}$$

1. **Give the formula for the profit function.**

   SOLUTION: **Profit** $= \mathbb{P}(x) = R(x) - C(x)$, where

   $$\begin{cases} R(x) = & \text{Revenue} = \text{Quantity } \times \text{Price} \\ C(x) = & \text{Cost function} \end{cases}$$

   Since the Quantity = is the number of shirts sold = x and the price $p = 30 - 0.2\sqrt{x}$, then

   $$\begin{cases} R(x) & = x \cdot (30 - 0.2\sqrt{x}) = 30x - 0.2x^{\frac{3}{2}} \\ C(x) & = \$500 + \$9 \cdot \text{number of shirts} = 500 + 9x \end{cases}$$

   Hence $\mathbb{P}(x) = 30x - 0.2x^{\frac{3}{2}} - (500 + 9x) = 21x - 0.2x^{\frac{3}{2}} - 500$.

2. **How many shirts should be produced to maximize profit?**

   <u>SOLUTION:</u> To maximize the $\mathbb{P}(x)$, we need to find the critical numbers:

   $\mathbb{P}'(x) = 21 - 0.3x^{\frac{1}{2}} = 0 \Rightarrow \sqrt{x} = \frac{21}{0.3} = 70$. Therefore, $\boxed{x = 4900}$ is the only critical number.

   Moreover, $\mathbb{P}''(x) = -0.15x^{-\frac{1}{2}} \Rightarrow \mathbb{P}''(4900) = -0.15\ 4900^{-\frac{1}{2}} = -\frac{0.15}{70} < 0$. By the second derivative test, R has a local maximum at $x = 4900$, which is an absolute maximum since it is the only critical number.

3. **At what price will the shirts be sold?**

   <u>SOLUTION:</u> The best price to maximize the profit is then: $\boxed{p(4900) = 30 - 0.2\sqrt{4900} = \$16}$.

4. **What is her resulting profit?**

   <u>SOLUTION:</u> The corresponding profit is $\boxed{\mathbb{P}(4900) = \$33800}$

Problem 5. Farmers can get 2 dollars per bushel for their potatoes on July 1, and after that, the price drops by 2 cents per bushel per extra day. On July 1, a farmer had 80 bushels of potatoes in the field and estimates that the crop is increasing at the rate of 1 bushel per day. When should the farmer harvest the potatoes to maximize his revenue?

- <u>SOLUTION:</u>

  Let **x**= the number of extra days after July 1.

  Let **R**= the revenue = quantity × price. Then:

  $$\begin{cases} \text{price} & = \$2 - x \cdot (\$0.02) = 2 - 0.02x \text{ dollars}, \\ \text{quantity} & = \text{number of bushels} = 80 + x \cdot 1 \text{ bushel per day}) = 80 + x \text{ bushels.} \end{cases}$$

  Hence $\boxed{R(x) = (80 + x)(2 - 0.02x) = 160 + 0.4x - 0.02x^2}$ to maximize ! Lets find the critical numbers:

  $R'(x) = 0.4 - 0.04x = 0 \Rightarrow x = \frac{0.4}{0.04} = 10$ is the only critical number.

  Moreover, $R''(x) = -0.04 \Rightarrow R''(10) = -0.04 < 0$. By the second derivative test, R has a local maximum at $\boxed{x = 10}$, which is an absolute maximum since it is the only critical number.

  The farmer should harvest the potatoes 10 extra days after July 1, so on $\boxed{\text{July } 11}$.

Problem 6. A landscape architect plans to enclose a 3000 square foot rectangular region in a botanical garden, She will use shrubs costing $25 per foot along three sides and fencing costing $10 per foot along the fourth side, Find the minimum total cost.

- <u>SOLUTION:</u> If the rectangular region has dimensions x and y, then its area is $A = xy = 3000ft^2$. So $\boxed{y = \frac{3000}{x}}$.

  If **y** is the side with fencing costing $10 per foot, then the cost for this side is **$ 10 y**.

  The cost for the three other sides, where shrubs costing $15 is used, is then **$ 15 (2x+y)**.

  Therefore the total cost is: $C(x) = 10y + 15(2x + y) = 30x + 25y$.

  Since $y = \frac{3000}{x}$, then $C(x) = 30x + 25\frac{3000}{x}$ that we wish to minimize.

  Since $C'(x) = 30 - 25\frac{3000}{x^2}$, then $C'(x) = 0$ for $x^2 = 25\frac{3000}{30} = 2500$. Therefore, since x is positive, we have only one critical number in the domain which is $\boxed{x = 50ft}$.

  Since $C''(x) = 25\frac{1500}{x^3}$, we have $C''(50) > 0$. Thus, by the 2^{nd} *derivative test*, C has a local minimum at $x = 50$, and therefore an absolute minimum because we have only one critical number in the domain. Hence, the minimum cost is $C(50) = \$4500$, with the dimensions $x = 50$ ft and $y = \frac{3000}{50} = 60$ ft.

Maximum−Minimum Problems: Optimization

To optimize a function means the following:
To maximize the revenue function
To minimize the cost function
To maximize the profit function.
Procedure:

(a) Define a variable x and build the equation of a function based on the information given in the problem.

(b) Find the derivative of that function to get the critical number.

(c) Test the C.N. using the first or second derivative test.

(d) Answer any question given in the problem.

Example 4: A manufacturer sells 500 units per week at 31 dollars per unit.
If the price is reduced by one dollar, 20 more units will be sold.

To maximize the revenue, find:

(a) the selling price
let x be the number of one dollar reduction
price in dollars per unit: $31 - x$

(b) the number of units sold
number of units sold: $500 + 20x$

(c) the maximum revenue
Revenue = (price per unit) . (number of units)
$R(x) = (31 - x)\,.\,(500 + 20x) = -20x^2 + 120x + 15500$

Once the equation of the revenue function is found, use the first derivative to find C.N., test the C.N. using the first or second derivative and then answer the following questions:
(1) find the selling price to maximize the revenue;
(2) find the number of units sold to maximize the revenue;
(3) find the maximum revenue.

$R'(x) = -40x + 120 \implies R'(x) = 0 \implies -40x + 120 = 0 \implies x = 3$

Test the critical number $x = 3$ with the second derivative: $R''(x) = -40 < 0$, relative maximum

(1) the selling price to maximize the revenue is $31 - 3 = 28$ dollars per unit
(2) the number of units sold to maximize the revenue is $500 + 20(3) = 560$ units
(3) the maximum revenue is $R(3) = (28)\,.\,(560) = \$15\,680$

Minimizing Inventory Costs

Example:

A retail appliance store sells **2500** TV sets per year. It costs **$10** to store one set for a year. To reorder, there is a fixed cost of **$20** to cover administrative costs per order, plus **$9** shipping fee for each set ordered.

a. How many times per year should the store reorder to minimize inventory costs?

Solution:

Let x = number of items per order

Yearly carrying cost = (yearly storage cost per item)•(average number of items carried)

$$= 10 \bullet \frac{x}{2} = 5x$$

Yearly reordering cost = (cost of each order)•(number of orders placed per year)

$$= (20 + 9x) \bullet \frac{2500}{x} = \frac{50000}{x} + 22500$$

Total Inventory Cost (y) = (yearly carrying cost) + (yearly reordering cost)

$$y = (5x) + \left(\frac{50000}{x} + 22500\right)$$

$$y' = 5 - \frac{50000}{x^2}$$

$y' = 0$ when

$$5 = \frac{50000}{x^2}$$

$$x^2 = 10000$$

$$x = 100$$

$y'' = \frac{50000}{x^3} > 0$ when x = 100, so the minimum number of orders placed per year $= \frac{2500}{100} = 25$ orders.

b. How many sets should be ordered each time?

Solution:

x = 100, so each order should contain 100 TV sets.

When a firm orders and stores their inventory, a decision must be made about how much to order and how often. Ordering costs must be analyzed against carrying costs, which are associated with storing, insurance, tied up capital, etc...

A firm can apply optimization to the situation.

A mattress store expects to sell 490 mattresses at a steady rate next year. The store manager plans to order from the manufacturer by placing several orders of the same size spaced equally throughout the year. The ordering costs 120 dollars and the storage costs, based on the average number of mattresses, is 24 dollars per mattress. Minimize inventory costs.

Let x be the number of mattresses in each order and r be the number of orders per year. Firstly, $xr = 490$. Why? An equation involving the variables is a constraint on those variables.

Note that on average $\frac{x}{2}$ mattresses are being stored.

The cost is given by

$$C = 24 \cdot \frac{x}{2} + 120r.$$

Then

$$C(x) = 12x + 120 \cdot \frac{490}{x}.$$

$$C'(x) = 12 - 120 \cdot \frac{490}{x^2} = 0.$$

$$12 = 120 \cdot \frac{490}{x^2}.$$

$$x^2 = 4900 \rightarrow x = 70.$$

So, $r = 7$ orders of 70 mattresses per order. This minimizes cost. Just look at

$$C''(x) = 240 \cdot \frac{490}{x^3} \text{ which is positive for positive } x.$$

How about minimizing construction costs?

A supermarket is to be designed as a rectangular building with a floor area of $12,000$ square feet. The front will be mostly glass and cost 70 dollars per running foot. The other three walls cost 50 dollars per running foot. Find the dimensions of the base that minimize construction costs.

Let w represent the length of the front and back walls. Let ℓ represent the side wall lengths. Then $\ell w = 12,000$ is a constraint. The cost is given by

$$C = 70w + 50(2\ell + w).$$

So,

$$C(w) = 120w + 100 \cdot \frac{12,000}{w}.$$

$$C'(w) = 120 - 100 \cdot \frac{12,000}{w^2} = 0.$$

$$w^2 = 10,000 \rightarrow w = 100 \text{ feet}.$$

So the supermarket should be built 100 feet across the front and 120 feet deep (side walls). Again the second derivative is positive.

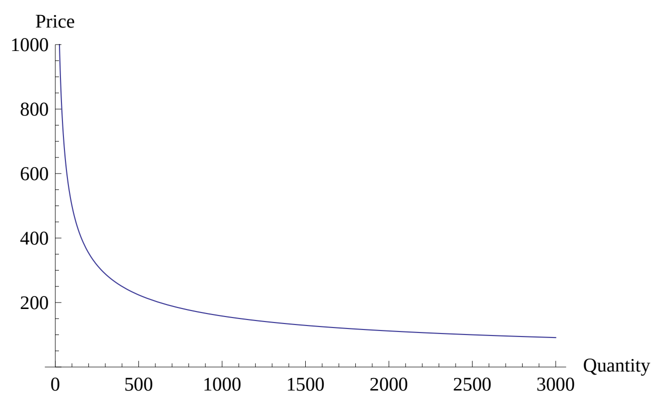
Business Applications:

In business, cost, revenue and profit are intimately related:

$$\text{profit} \;=\; \text{revenue} \;-\; \text{cost}.$$

As $C'(x)$ is the marginal cost, we call $R'(x)$ and $P'(x)$ marginal revenue and marginal profit respectively.

A demand curve for a good, $p = f(x)$, is the highest price that can be set in order to sell all x units of the good. Typically the price has to be set lower to sell larger numbers of units. For example, suppose the following is the demand curve for a certain store's signature sofas:



It looks like if the store wants to sell 2,000 sofas that it should set a price of roughly 100 dollars.

However, if the store only wants to sell 1000 sofas then it should set a price of roughly 150 dollars.

Of course, the revenue is given by $R(x) = xf(x)$, that is, the product of the price and the quantity.