

**Solve the problem.**

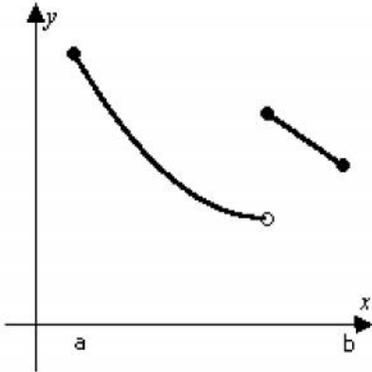
- 1) Water is falling on a surface, wetting a circular area that is expanding at a rate of  $4 \text{ mm}^2/\text{s}$ . How fast is the radius of the wetted area expanding when the radius is  $163 \text{ mm}$ ? (Round your answer to four decimal places.)  
 A)  $0.0039 \text{ mm/s}$                       B)  $0.0078 \text{ mm/s}$                       C)  $256.0396 \text{ mm/s}$                       D)  $0.0245 \text{ mm/s}$
- 2) A company knows that the unit cost  $C$  and the unit revenue  $R$  from the production and sale of  $x$  units are related by  $C = \frac{R^2}{206,000} + 11,857$ . Find the rate of change of unit revenue when the unit cost is changing by  $\$9/\text{unit}$  and the unit revenue is  $\$4000$ .  
 A)  $\$180.00/\text{unit}$                       B)  $\$708.73/\text{unit}$                       C)  $\$1185.70/\text{unit}$                       D)  $\$231.75/\text{unit}$

**Solve the problem. Round your answer, if appropriate.**

- 3) Water is discharged from a pipeline at a velocity  $v$  (in  $\text{ft}/\text{sec}$ ) given by  $v = 2000p^{(1/2)}$ , where  $p$  is the pressure (in  $\text{psi}$ ). If the water pressure is changing at a rate of  $0.450 \text{ psi}/\text{sec}$ , find the acceleration ( $dv/dt$ ) of the water when  $p = 56.0 \text{ psi}$ .  
 A)  $134 \text{ ft}/\text{sec}^2$                       B)  $60.1 \text{ ft}/\text{sec}^2$                       C)  $3370 \text{ ft}/\text{sec}^2$                       D)  $74.8 \text{ ft}/\text{sec}^2$
- 4) One airplane is approaching an airport from the north at  $184 \text{ km}/\text{hr}$ . A second airplane approaches from the east at  $210 \text{ km}/\text{hr}$ . Find the rate at which the distance between the planes changes when the southbound plane is  $33 \text{ km}$  away from the airport and the westbound plane is  $22 \text{ km}$  from the airport.  
 A)  $-405 \text{ km}/\text{hr}$                       B)  $-135 \text{ km}/\text{hr}$                       C)  $-540 \text{ km}/\text{hr}$                       D)  $-270 \text{ km}/\text{hr}$
- 5) A man  $6 \text{ ft}$  tall walks at a rate of  $3 \text{ ft}/\text{sec}$  away from a lamppost that is  $21 \text{ ft}$  high. At what rate is the length of his shadow changing when he is  $30 \text{ ft}$  away from the lamppost? (Do not round your answer)  
 A)  $\frac{6}{5} \text{ ft}/\text{sec}$                       B)  $\frac{1}{3} \text{ ft}/\text{sec}$                       C)  $\frac{2}{3} \text{ ft}/\text{sec}$                       D)  $15 \text{ ft}/\text{sec}$
- 6) The volume of a sphere is increasing at a rate of  $5 \text{ cm}^3/\text{sec}$ . Find the rate of change of its surface area when its volume is  $\frac{500\pi}{3} \text{ cm}^3$ . (Do not round your answer.)  
 A)  $10\pi \text{ cm}^2/\text{sec}$                       B)  $\frac{4}{3} \text{ cm}^2/\text{sec}$                       C)  $2 \text{ cm}^2/\text{sec}$                       D)  $\frac{125}{3} \text{ cm}^2/\text{sec}$
- 7) The radius of a right circular cylinder is increasing at the rate of  $4 \text{ in.}/\text{sec}$ , while the height is decreasing at the rate of  $9 \text{ in.}/\text{sec}$ . At what rate is the volume of the cylinder changing when the radius is  $16 \text{ in.}$  and the height is  $8 \text{ in.}$ ?  
 A)  $-1792 \text{ in.}^3/\text{sec}$                       B)  $-1792\pi \text{ in.}^3/\text{sec}$                       C)  $80 \text{ in.}^3/\text{sec}$                       D)  $-1280\pi \text{ in.}^3/\text{sec}$

Determine from the graph whether the function has any absolute extreme values on the interval  $[a, b]$ .

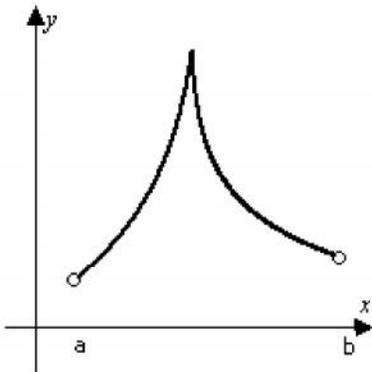
8)



- A) Absolute minimum and absolute maximum.
- C) Absolute maximum only.

- B) Absolute minimum only.
- D) No absolute extrema.

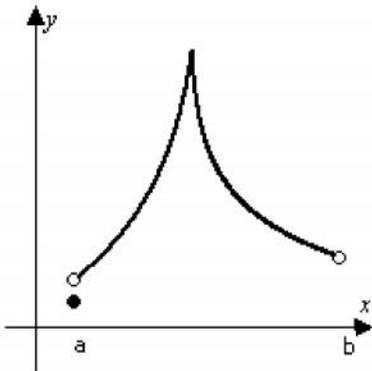
9)



- A) Absolute minimum and absolute maximum.
- C) Absolute minimum only.

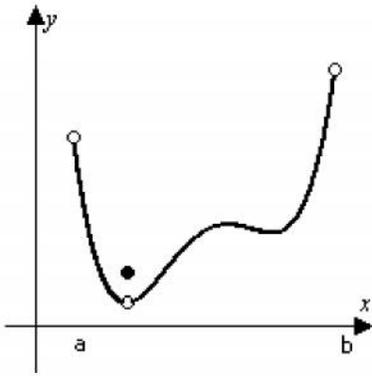
- B) No absolute extrema.
- D) Absolute maximum only.

10)



- A) No absolute extrema.
- C) Absolute maximum only.

- B) Absolute minimum and absolute maximum.
- D) Absolute minimum only.



11)

- A) Absolute minimum only.  
 C) Absolute minimum and absolute maximum.

- B) Absolute maximum only.  
 D) No absolute extrema.

**Determine all critical points for the function.**

12)  $f(x) = x^3 - 6x^2 + 4$

A)  $x = 0$  and  $x = 2$

B)  $x = 0$  and  $x = 4$

C)  $x = -2$  and  $x = 2$

D)  $x = 0$

13)  $f(x) = 20x^3 - 3x^5$

A)  $x = 2$

C)  $x = -2$

B)  $x = 0, x = -2, \text{ and } x = 2$

D)  $x = -2$  and  $x = 2$

14)  $f(x) = \frac{-6x}{x+5}$

A)  $x = 5$

B)  $x = -5$

C)  $x = -30$  and  $x = 0$

D)  $x = 0$  and  $x = -5$

15)  $f(x) = (x - 4)^7$

A)  $x = 4$

C)  $x = 4$  and  $x = 7$

B)  $x = 0$  and  $x = 4$

D)  $x = 0, x = 4, \text{ and } x = 7$

16)  $y = 2x^2 - 64\sqrt{x}$

A)  $x = 0, x = 4, \text{ and } x = -4$

C)  $x = 0$

B)  $x = 4$

D)  $x = 0$  and  $x = 4$

**Find the absolute extreme values of the function on the interval.**

17)  $g(x) = -x^2 + 11x - 24, 3 \leq x \leq 8$

A) absolute maximum is  $\frac{29}{4}$  at  $x = \frac{13}{2}$ ; absolute minimum is 0 at 8 and 0 at  $x = 3$

B) absolute maximum is  $\frac{25}{4}$  at  $x = \frac{11}{2}$ ; absolute minimum is 0 at 8 and 0 at  $x = 3$

C) absolute maximum is  $\frac{217}{4}$  at  $x = \frac{11}{2}$ ; absolute minimum is 0 at 8 and 0 at  $x = 3$

D) absolute maximum is  $\frac{25}{4}$  at  $x = \frac{13}{2}$ ; absolute minimum is 0 at 8 and 0 at  $x = 3$

18)  $f(\theta) = \sin\left(\theta + \frac{\pi}{2}\right), 0 \leq \theta \leq \frac{7\pi}{4}$

- A) absolute maximum is 1 at  $\theta = 0$ ; absolute minimum is -1 at  $\theta = \pi$
- B) absolute maximum is 1 at  $\theta = \frac{3}{2}\pi$ ; absolute minimum is -1 at  $\theta = \frac{1}{2}\pi$ ,
- C) absolute maximum is 1 at  $\theta = \frac{1}{2}\pi$ ; absolute minimum is -1 at  $\theta = \frac{1}{2}\pi$
- D) absolute maximum is 1 at  $\theta = \frac{1}{2}\pi$ ; absolute minimum is -1 at  $\theta = \frac{1}{2}\pi$ ,

19)  $f(x) = \csc x, -\frac{\pi}{2} \leq x \leq \frac{3\pi}{2}$

- A) absolute maximum is -1 at  $x = \pi$ ; absolute minimum is 1 at  $x = 0$
- B) absolute maximum is 1 at  $x = \pi$ ; absolute minimum is -1 at  $x = \pi$
- C) absolute maximum does not exist; absolute minimum does not exist
- D) absolute maximum is 0 at  $x = -\pi$ ; absolute minimum is -1 at  $x = \pi$

20)  $F(x) = -\frac{2}{x^2}, 0.5 \leq x \leq 2$

- A) absolute maximum is  $-\frac{1}{2}$  at  $x = 2$ ; absolute minimum is -8 at  $x = -\frac{1}{2}$
- B) absolute maximum is  $-\frac{1}{2}$  at  $x = 2$ ; absolute minimum is -8 at  $x = \frac{1}{2}$
- C) absolute maximum is  $\frac{1}{2}$  at  $x = \frac{1}{2}$ ; absolute minimum is -8 at  $x = 2$
- D) absolute maximum is  $-\frac{1}{2}$  at  $x = \frac{1}{2}$ ; absolute minimum is -8 at  $x = -2$

21)  $F(x) = \sqrt[3]{x}, -3 \leq x \leq 64$

- A) absolute maximum is 4 at  $x = 64$ ; absolute minimum is -4 at  $x = -64$
- B) absolute maximum is 0 at  $x = 0$ ; absolute minimum is 4 at  $x = 64$
- C) absolute maximum is 4 at  $x = -64$ ; absolute minimum is 0 at  $x = 0$
- D) absolute maximum is 4 at  $x = 64$ ; absolute minimum is 0 at  $x = 0$

22)  $g(x) = 6 - 5x^2, -2 \leq x \leq 3$

- A) absolute maximum is 12 at  $x = 0$ ; absolute minimum is -14 at  $x = 3$
- B) absolute maximum is 6 at  $x = 0$ ; absolute minimum is -39 at  $x = 3$
- C) absolute maximum is 5 at  $x = 0$ ; absolute minimum is -51 at  $x = 3$
- D) absolute maximum is 30 at  $x = 0$ ; absolute minimum is -14 at  $x = -2$

Find the absolute extreme values of the function on the interval.

23)  $f(x) = \ln(x + 2) + \frac{1}{x}, 1 \leq x \leq 9$

- A) absolute minimum value is  $-1$  at  $x = -1$ ; absolute maximum value is  $\ln 11 + \frac{1}{9}$  at  $x = 9$
- B) absolute minimum value is  $\ln 3 + 1$  at  $x = 1$ ; absolute maximum value is  $\ln 11 + \frac{1}{9}$  at  $x = 9$
- C) absolute minimum value is  $\ln 4 + \frac{1}{2}$  at  $x = 2$ ; absolute maximum value is  $\ln 3 + 1$  at  $x = 1$
- D) absolute minimum value is  $\ln 4 + \frac{1}{2}$  at  $x = 2$ ; absolute maximum value is  $\ln 11 + \frac{1}{9}$  at  $x = 9$

24)  $f(x) = e^x - x, -4 \leq x \leq 2$

- A) absolute minimum value is  $1$  at  $x = 0$ ; absolute maximum value is  $e^{-4} + 4$  at  $x = -4$
- B) absolute minimum value is  $1$  at  $x = 0$ ; no maximum value
- C) absolute minimum value is  $1$  at  $x = 0$ ; absolute maximum value is  $e^2 - 2$  at  $x = 2$
- D) absolute minimum value is  $e^{-4} + 4$  at  $x = -4$ ; absolute maximum value is  $e^2 - 2$  at  $x = 2$

Find the extreme values of the function and where they occur.

25)  $y = x^3 - 12x + 2$

- A) None
- B) Local maximum at  $(2, -14)$ , local minimum at  $(-2, 18)$ .
- C) Local maximum at  $(0, 0)$ .
- D) Local maximum at  $(-2, 18)$ , local minimum at  $(2, -14)$ .

26)  $y = x^3 - 3x^2 + 5x - 6$

- A) Absolute maximum is  $2$  at  $x = 1$ .
- B) None
- C) Absolute minimum is  $2$  at  $x = -1$ .
- D) Absolute maximum is  $2$  at  $x = 2$ .

27)  $y = \frac{x + 1}{x^2 + 3x + 3}$

- A) None
- B) Absolute maximum is  $-\frac{1}{3}$  at  $x = 0$ ; absolute minimum is  $1$  at  $x = -2$ .
- C) Absolute maximum is  $\frac{1}{3}$  at  $x = 0$ ; absolute minimum is  $-1$  at  $x = -2$ .
- D) Absolute maximum is  $3$  at  $x = 0$ ; absolute minimum is  $\frac{1}{3}$  at  $x = -2$ .

28)  $y = x^2e^{-x} + 2xe^{-x}$

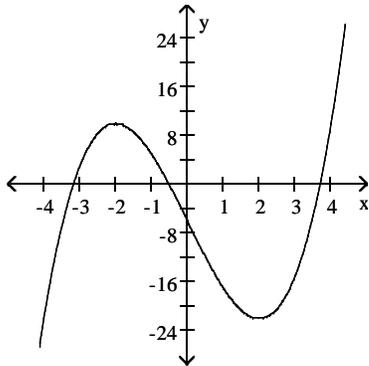
- A) Absolute maximum value is  $2e^{-\sqrt{2}}(1 + \sqrt{2})$  at  $x = \sqrt{2}$ ; no minimum value.
- B) Absolute minimum value is  $2e^{\sqrt{2}}(1 - \sqrt{2})$  at  $x = -\sqrt{2}$ ; maximum value is  $2e^{-\sqrt{2}}(1 + \sqrt{2})$  at  $x = \sqrt{2}$ .
- C) Absolute minimum value is  $2e^{\sqrt{2}}(1 - \sqrt{2})$  at  $x = -\sqrt{2}$ ; no maximum value.
- D) None

**Solve the problem.**

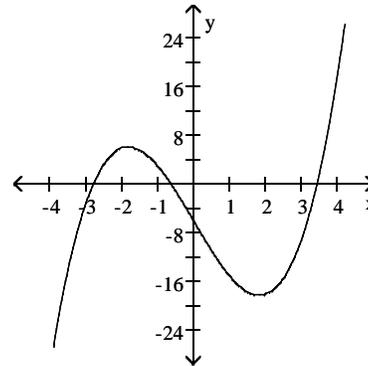
29) Using the following properties of a twice-differentiable function  $y = f(x)$ , select a possible graph of  $f$ .

$x$	$y$	Derivatives
$x < 2$		$y' > 0, y'' < 0$
-2	10	$y' = 0, y'' < 0$
$-2 < x < 0$		$y' < 0, y'' < 0$
0	-6	$y' < 0, y'' = 0$
$0 < x < 2$		$y' < 0, y'' > 0$
2	-22	$y' = 0, y'' > 0$
$x > 2$		$y' > 0, y'' > 0$

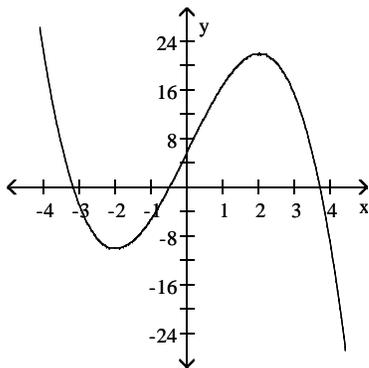
A)



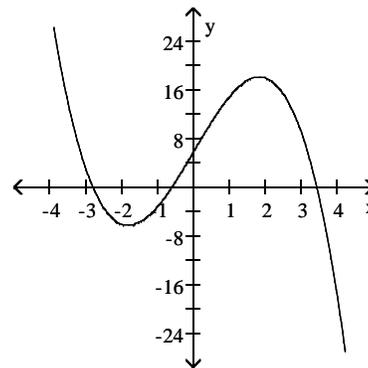
B)



C)



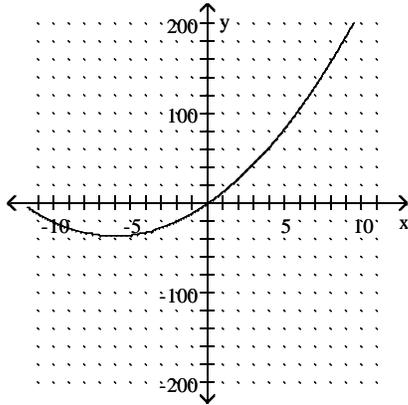
D)



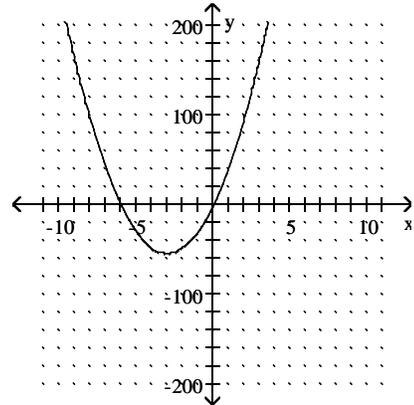
Graph the equation. Include the coordinates of any local extreme points and inflection points.

30)  $y = 6x^2 + 36x$

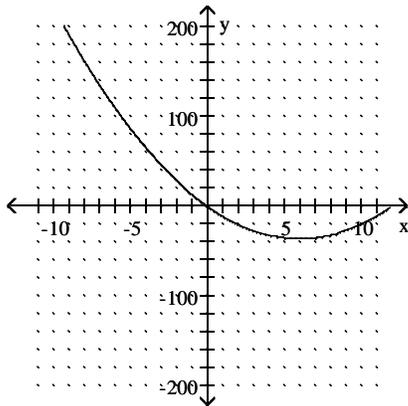
A) local minimum:  $(-6, -36)$   
no inflection points



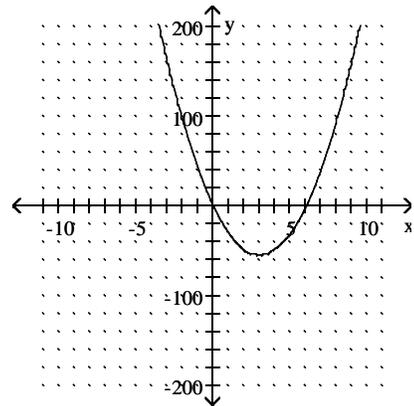
B) local minimum:  $(-3, -54)$   
no inflection points



C) local minimum:  $(6, -36)$   
no inflection points

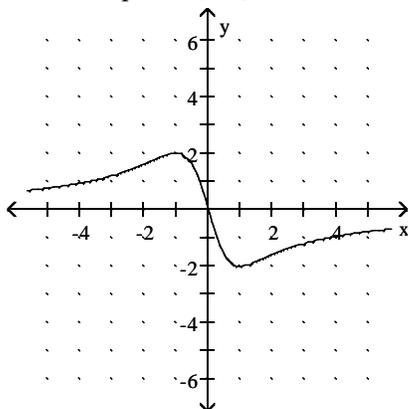


D) local minimum:  $(3, -54)$   
no inflection points

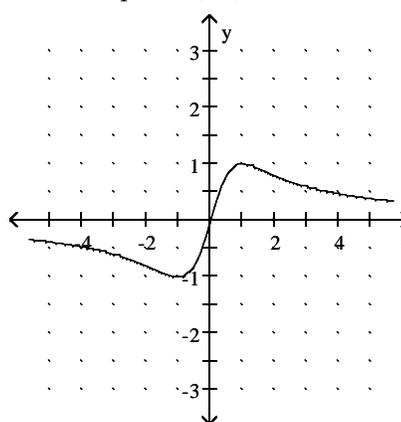


$$31) y = \frac{4x}{x^2 + 1}$$

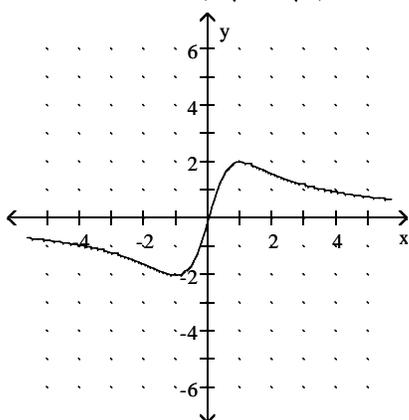
- A) local minimum:  $(1, -2)$   
 local maximum:  $(-1, 2)$   
 inflection point:  $(0, 0)$



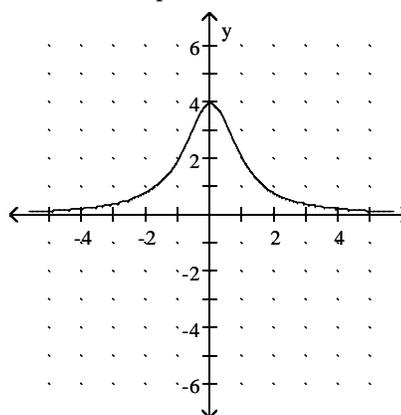
- B) local minimum:  $(-1, -1)$   
 local maximum:  $(1, 1)$   
 inflection point:  $(0, 0)$



- C) local minimum:  $(-1, -2)$   
 local maximum:  $(1, 2)$   
 inflection points:  $(0, 0)$ ,  $(-1\sqrt{3}, -1\sqrt{3})$ ,  
 $(1\sqrt{3}, 1\sqrt{3})$



- D) absolute maximum:  $(0, 4)$   
 no inflection point



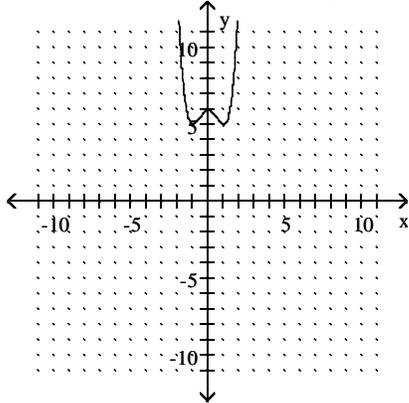
Sketch the graph and show all local extrema and inflection points.

32)  $y = -x^4 + 2x^2 - 6$

A) Local minima:  $(-1, 5), (1, 5)$

Local maximum:  $(0, 6)$

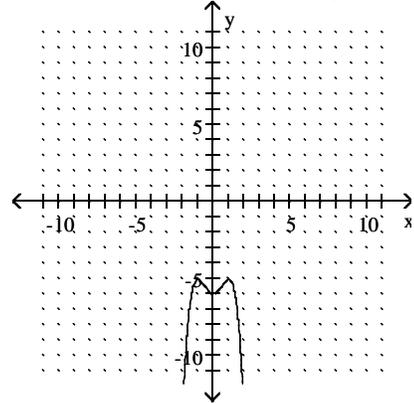
Inflection point:  $\left(-\sqrt{\frac{1}{3}}, \frac{49}{9}\right), \left(\sqrt{\frac{1}{3}}, \frac{49}{9}\right)$



B) Local maxima:  $(-1, -5), (1, -5)$

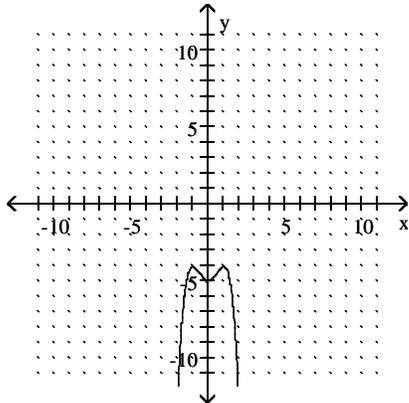
Local minimum:  $(0, -6)$

Inflection points:  $\left(-\sqrt{\frac{1}{3}}, \frac{5}{9}\right), \left(\sqrt{\frac{1}{3}}, \frac{5}{9}\right)$



C) Local maxima:  $(-1, -5), (1, -5)$

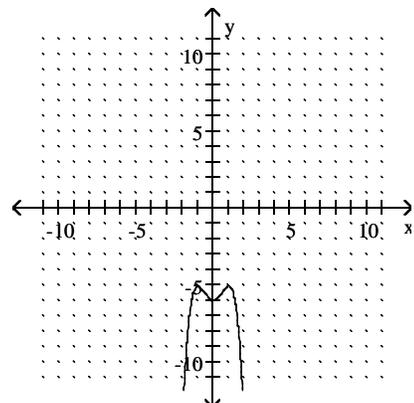
Inflection points:  $\left(-\sqrt{\frac{1}{3}}, \frac{5}{9}\right), \left(\sqrt{\frac{1}{3}}, \frac{5}{9}\right)$



D) Local maxima:  $(-1, -5), (1, -5)$

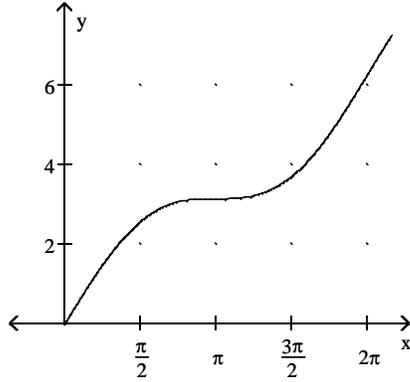
Local minimum:  $(0, -6)$

No inflection points

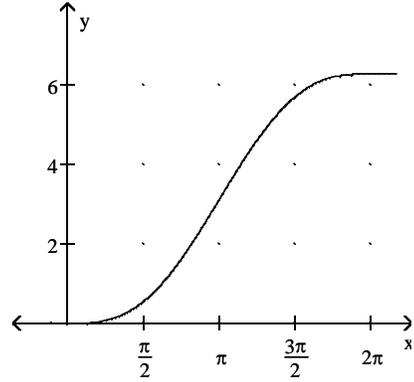


33)  $y = x + \sin x, 0 \leq x \leq 2\pi$

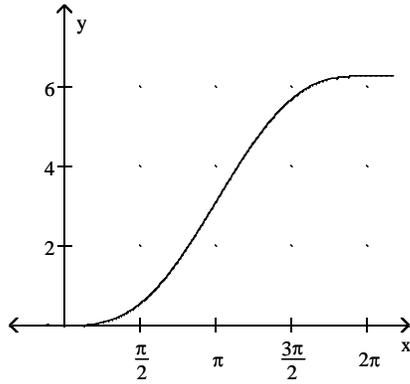
- A) Local minimum:  $(0, 0)$   
 Local maximum:  $(2\pi, 2\pi)$   
 Inflection point:  $(\pi, \pi)$



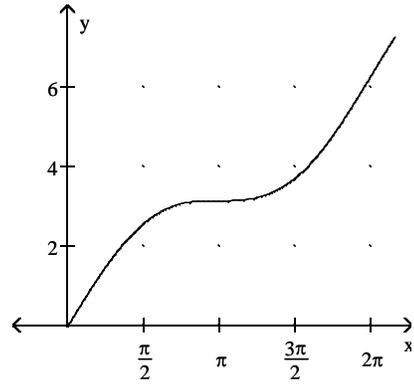
- B) Local minimum:  $(0, 0)$   
 Local maximum:  $(2\pi, 2\pi)$   
 Inflection point:  $(\pi, \pi)$



- C) Local minimum:  $(0, 0)$   
 Local maximum:  $(2\pi, 2\pi)$   
 No inflection points



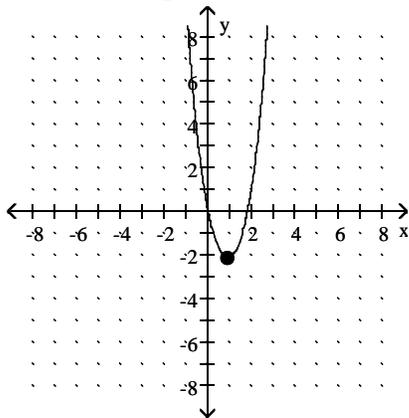
- D) Local minimum:  $(0, 0)$   
 Local maximum:  $(2\pi, 2\pi)$   
 No inflection points



34)  $y = e^x - 6e^{-x} - 7x$

A) Local minimum  $\left(\frac{1}{2} \ln 6, -\frac{7}{2} \ln 6\right)$

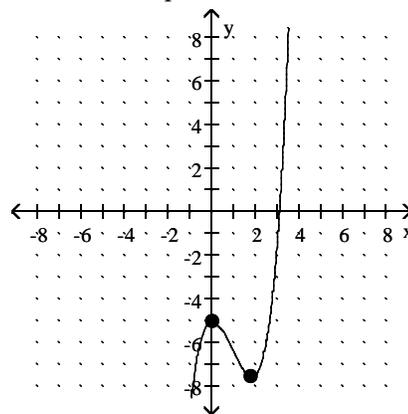
No inflection point



B) Local maximum  $(0, -5)$

Local minimum  $(\ln 6, 5 - 7 \ln 6)$

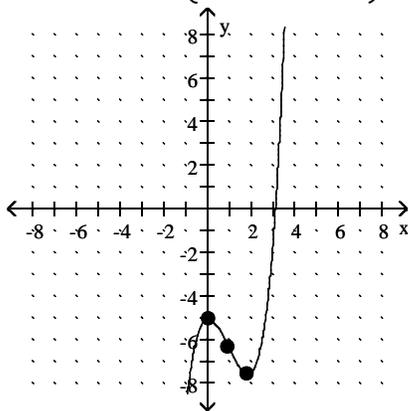
No inflection point



C) Local maximum  $(0, -5)$

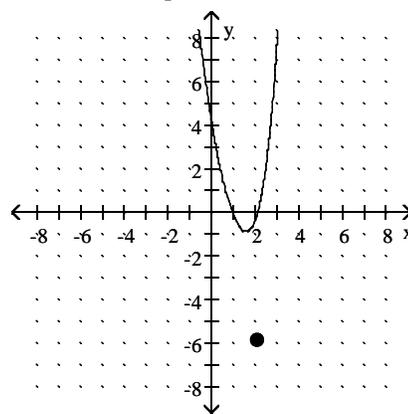
Local minimum  $(\ln 6, 5 - 7 \ln 6)$

Inflection point  $\left(\frac{1}{2} \ln 6, -\frac{7}{2} \ln 6\right)$



D) Local minimum  $(2, -6)$

No inflection point



**Solve the problem.**

35) From a thin piece of cardboard 50 in. by 50 in., square corners are cut out so that the sides can be folded up to make a box. What dimensions will yield a box of maximum volume? What is the maximum volume? Round to the nearest tenth, if necessary.

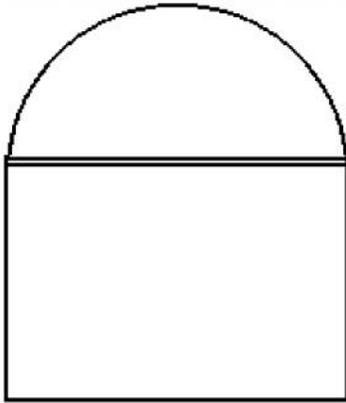
A) 25 in.  $\times$  25 in.  $\times$  12.5 in.;  $7812.5 \text{ in}^3$

B) 33.3 in.  $\times$  33.3 in.  $\times$  8.3 in.;  $9259.3 \text{ in}^3$

C) 33.3 in.  $\times$  33.3 in.  $\times$  16.7 in.;  $18,518.5 \text{ in}^3$

D) 16.7 in.  $\times$  16.7 in.  $\times$  16.7 in.;  $4629.6 \text{ in}^3$

- 36) A window is in the form of a rectangle surmounted by a semicircle. The rectangle is of clear glass, whereas the semicircle is of tinted glass that transmits only one-third as much light per unit area as clear glass does. The total perimeter is fixed. Find the proportions of the window that will admit the most light. Neglect the thickness of the frame.



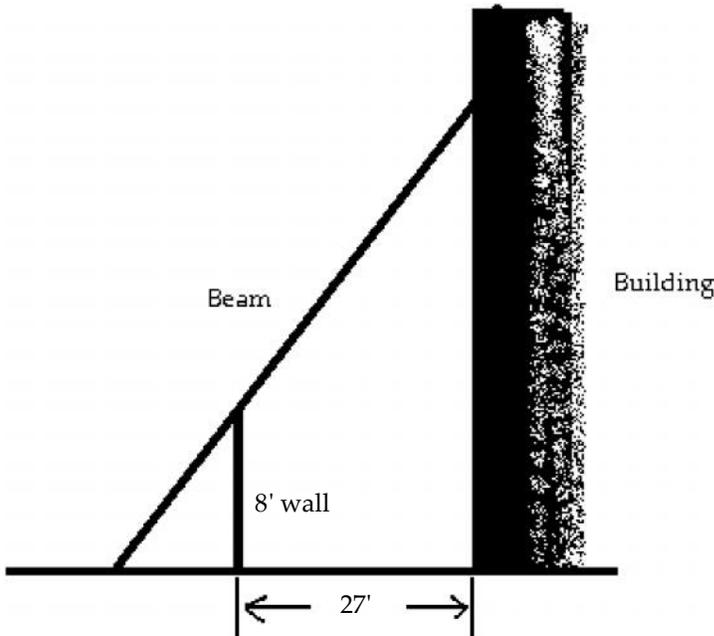
A)  $\frac{\text{width}}{\text{height}} = \frac{12}{6 + 2\pi}$

B)  $\frac{\text{width}}{\text{height}} = \frac{3}{6 + 2\pi}$

C)  $\frac{\text{width}}{\text{height}} = \frac{12}{3 + 2\pi}$

D)  $\frac{\text{width}}{\text{height}} = \frac{12}{6 + \pi}$

- 37) The 8 ft wall shown here stands 27 feet from the building. Find the length of the shortest straight beam that will reach to the side of the building from the ground outside the wall.



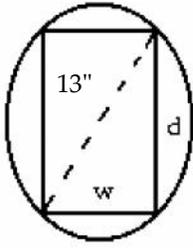
A) 45.9 ft

B) 46.9 ft

C) 47.9 ft

D) 35 ft

- 38) The strength  $S$  of a rectangular wooden beam is proportional to its width times the square of its depth. Find the dimensions of the strongest beam that can be cut from a 13-in.-diameter cylindrical log. (Round answers to the nearest tenth.)



- A)  $w = 8.5$  in.;  $d = 11.6$  in.                      B)  $w = 6.5$  in.;  $d = 11.6$  in.  
 C)  $w = 7.5$  in.;  $d = 10.6$  in.                      D)  $w = 8.5$  in.;  $d = 9.6$  in.
- 39) If the price charged for a candy bar is  $p(x)$  cents, then  $x$  thousand candy bars will be sold in a certain city, where  $p(x) = 63 - \frac{x}{18}$ . How many candy bars must be sold to maximize revenue?
- A) 567 candy bars    B) 1134 candy bars  
 C) 567 thousand candy bars                              D) 1134 thousand candy bars
- 40) A rectangular field is to be enclosed on four sides with a fence. Fencing costs \$4 per foot for two opposite sides, and \$6 per foot for the other two sides. Find the dimensions of the field of area  $800 \text{ ft}^2$  that would be the cheapest to enclose.
- A) 23.1 ft @ \$4 by 34.6 ft @ \$6                      B) 42.4 ft @ \$4 by 18.9 ft @ \$6  
 C) 18.9 ft @ \$4 by 42.4 ft @ \$6                      D) 34.6 ft @ \$4 by 23.1 ft @ \$6

**Find the linearization  $L(x)$  of  $f(x)$  at  $x = a$ .**

- 41)  $f(x) = 4x^2 - 2x - 4$ ,  $a = 3$   
 A)  $L(x) = 26x - 40$                       B)  $L(x) = 26x + 32$                       C)  $L(x) = 22x - 40$                       D)  $L(x) = 22x + 32$
- 42)  $f(x) = \sqrt{3x + 49}$ ,  $a = 0$   
 A)  $L(x) = \frac{3}{14}x + 7$                       B)  $L(x) = \frac{3}{7}x - 7$                       C)  $L(x) = \frac{3}{7}x + 7$                       D)  $L(x) = \frac{3}{14}x - 7$
- 43)  $f(x) = \frac{x}{9x + 5}$ ,  $a = 0$   
 A)  $L(x) = -\frac{1}{25}x$                       B)  $L(x) = \frac{1}{25}x$                       C)  $L(x) = \frac{1}{5}x$                       D)  $L(x) = -\frac{1}{5}x$
- 44)  $f(x) = \sin x$ ,  $a = 0$   
 A)  $L(x) = 5x + 1$                       B)  $L(x) = 0$                       C)  $L(x) = -x$                       D)  $L(x) = x$

**Solve the problem.**

- 45)  $A = \pi r^2$ , where  $r$  is the radius, in centimeters. By approximately how much does the area of a circle decrease when the radius is decreased from 3.0 cm to 2.8 cm? (Use 3.14 for  $\pi$ .)
- A)  $1.9 \text{ cm}^2$                       B)  $3.6 \text{ cm}^2$                       C)  $4.0 \text{ cm}^2$                       D)  $3.8 \text{ cm}^2$

- 46)  $V = \frac{4}{3}\pi r^3$ , where  $r$  is the radius, in centimeters. By approximately how much does the volume of a sphere increase when the radius is increased from 3.0 cm to 3.2 cm? (Use 3.14 for  $\pi$ )
- A) 22.8 cm<sup>3</sup>                      B) 22.4 cm<sup>3</sup>                      C) 1.5 cm<sup>3</sup>                      D) 22.6 cm<sup>3</sup>

**Use l'Hopital's Rule to evaluate the limit.**

- 47)  $\lim_{x \rightarrow 0} \frac{\cos 3x - 1}{x^2}$
- A)  $\frac{9}{2}$                       B)  $-\frac{9}{2}$                       C)  $\frac{3}{2}$                       D) 0
- 48)  $\lim_{x \rightarrow \pi/3} \frac{\cos x - \frac{1}{2}}{x - \frac{\pi}{3}}$
- A)  $\frac{\sqrt{3}}{2}$                       B)  $-\sqrt{3}$                       C)  $-\frac{\sqrt{3}}{2}$                       D)  $\frac{\sqrt{2}}{2}$
- 49)  $\lim_{\theta \rightarrow 0} \frac{7 - 7\cos \theta}{\sin 2\theta}$
- A)  $\frac{7}{2}$                       B)  $\infty$                       C) 0                      D) 1
- 50)  $\lim_{x \rightarrow 0} \frac{\sin 5x}{\tan 2x}$
- A)  $\frac{2}{5}$                       B)  $\frac{5}{2}$                       C)  $-\frac{5}{2}$                       D) 0