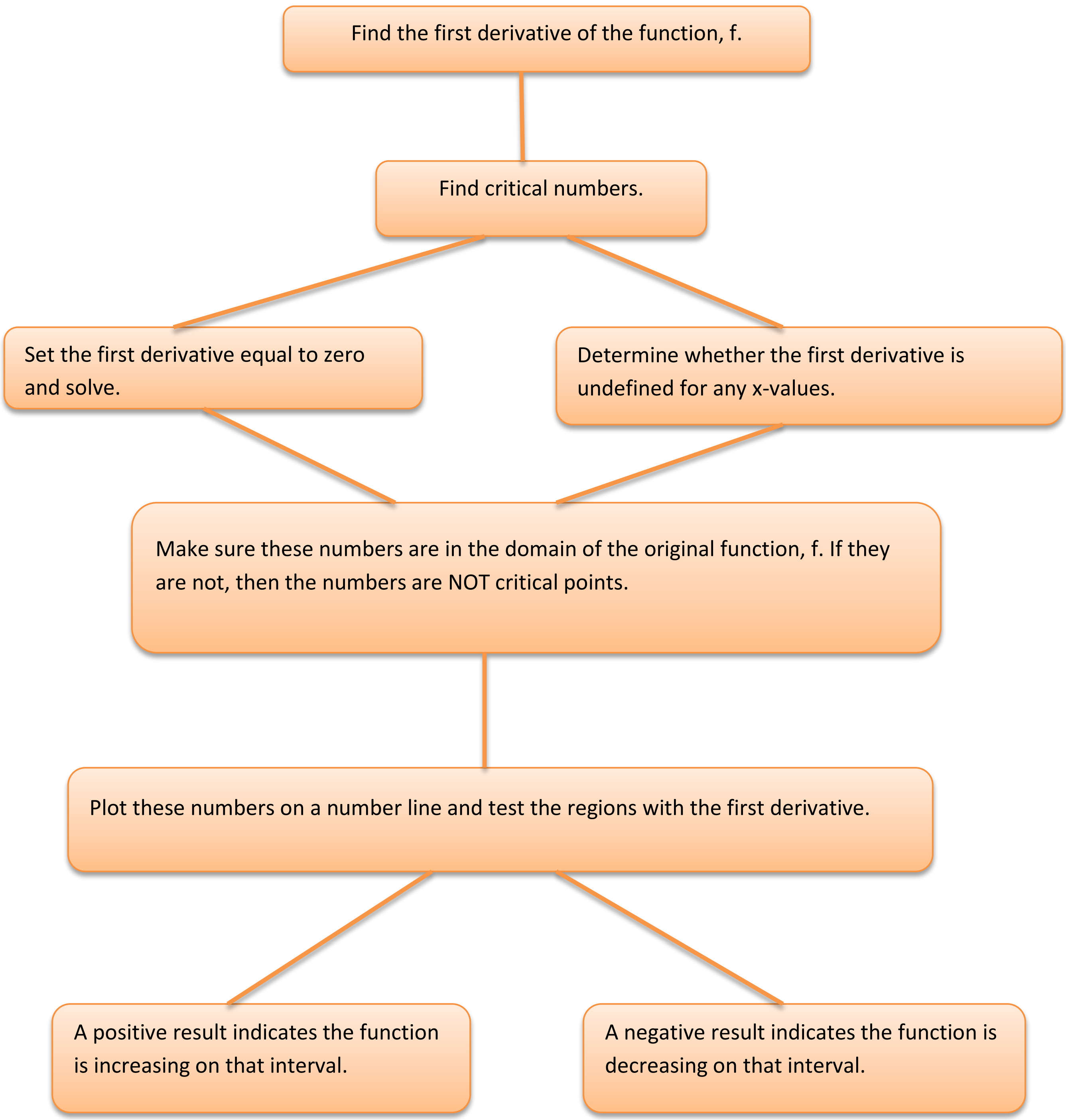
Intervals of Increasing & Decreasing



Example 1: Determine intervals on which function $\boldsymbol{f(x) = x^4 - 4x^3 + 4x^2}$ is <u>increasing or decreasing.</u>

$f'(x) = 4x^3 - 12x^2 + 8x$

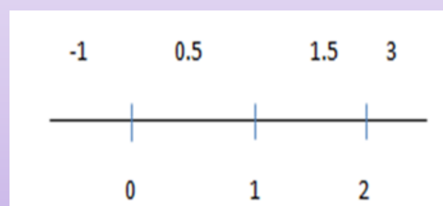
$f'(x) = 0$ ➔ $4x(x^2 - 3x + 2) = 0$

$4x(x-2)(x-1) = 0$ ➔ $x = 0, 1, 2$

The first derivative is defined for ALL x-values.

All critical numbers, namely x=0, 1, 2 are in the domain of the function f.

Plot the critical numbers on a number line and pick convenient numbers in those intervals. Start selecting numbers in the given intervals. Let's select a convenient number in the interval less than zero. How about -1? Then, select a number in the interval zero to 1, how about 0.5? Next, pick a number in the interval 1 to 2, let's say, 1.5. Finally select a number greater than 2, say, 3.



Test the regions with the <u>first derivative</u>:

$f'(-1) < 0$ ➔ f is decreasing on $(-\infty, 0)$

$f'\left(\frac{1}{2}\right) > 0$ ➔ f is increasing on $(0,1)$

$f'(1.5) < 0$ ➔ f is decreasing on $(1, 2)$

$f'(3) > 0$ ➔ f is increasing on $(2, \infty)$

$f'(3) > 0$ ➔ f is increasing on $(2, \infty)$

f is increasing on $(0,1) \cup (2, \infty)$

f is decreasing on $(-\infty, 0) \cup (1,2)$

Example 2: Determine the intervals on which the function $x\sqrt{x+3}$ is increasing or decreasing.

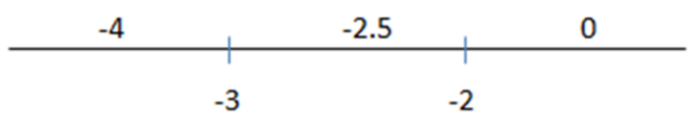
$$f'(x) = (x+3)^{\frac{1}{2}} + \tfrac{1}{2}x(x+3)^{-\frac{1}{2}}$$
$$= \frac{2(x+3)+x}{2\sqrt{x+3}}$$
$$= \frac{3x+6}{2\sqrt{x+3}}$$

$f'(x) = 0 \Rightarrow 3x + 6 = 0 \Rightarrow x = -2$

$f'(x)$ is undefined when $2\sqrt{x+3}$ =0. That is, $x = -3$.

Critical numbers $x = -3, -2$ are in the domain of function $f(x)$.

Plot the critical numbers on a number line and pick convenient numbers in those intervals. Start selecting numbers in the given intervals. Let's select a convenient number in the interval less than -3. How about -4? Then, select a number in the interval -3 to -2, how about -2.5? Finally pick a number in the interval greater than -2, say 0.



<u>Test the regions with the first derivative:</u>

$f'(-4)$ is not real

$f'(-2.5) < 0 \Rightarrow f$ is decreasing on $(-3, -2)$

$f'(0) > 0 \Rightarrow f$ is increasing on $(-2, \infty)$

f is increasing on $(-2, \infty)$

f is decreasing on $(-3, -2)$