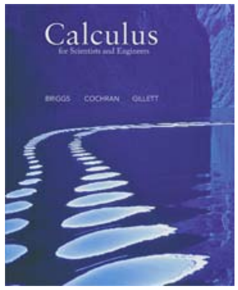
Chapter 1

Functions

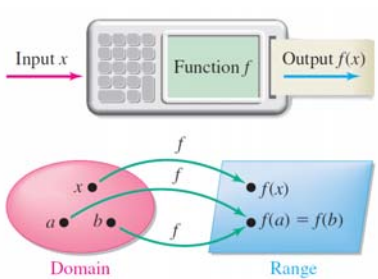


1.1

Review of Functions

1

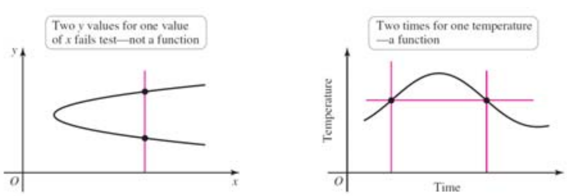
Figure 1.1



DEFINITION Function

A **function** f is a rule that assigns to each value x in a set D a *unique* value denoted $f(x)$. The set D is the **domain** of the function. The **range** is the set of all values of $f(x)$ produced as x varies over the domain (Figure 1.1).

Figure 1.2



2

Vertical Line Test

A graph represents a function if and only if it passes the **vertical line test**: Every vertical line intersects the graph at most once. A graph that fails this test does not represent a function.

Figure 1.3 (a)

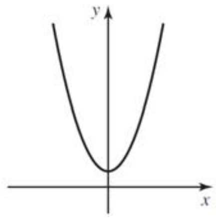


Figure 1.3 (b)

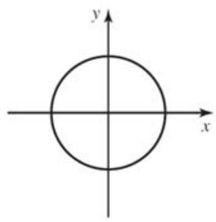


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Figure 1.3 (c)

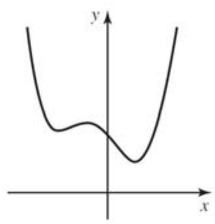


Figure 1.3 (d)

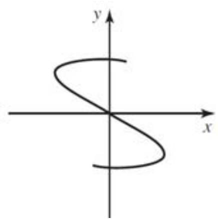


Figure 1.4

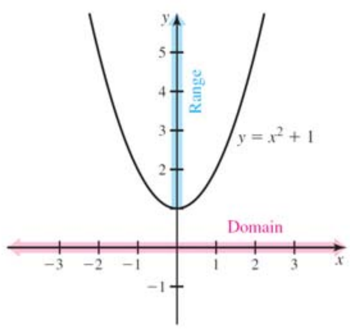


Figure 1.5

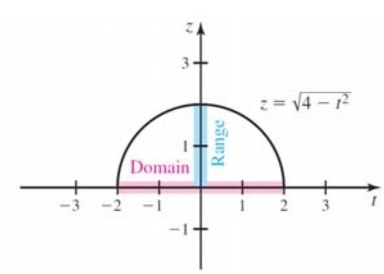


Figure 1.6

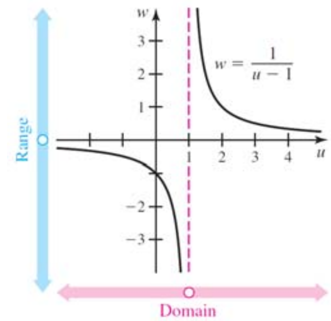
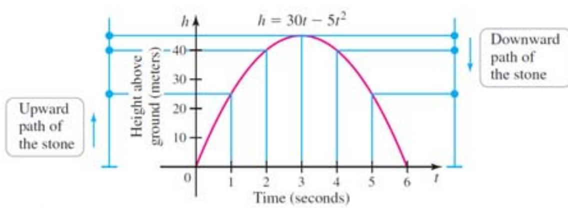


Figure 1.7



DEFINITION Composite Functions

Given two functions f and g, the composite function $f \circ g$ is defined by $(f \circ g)(x) = f(g(x))$. It is evaluated in two steps: $y = f(u)$, where $u = g(x)$. The domain of $f \circ g$ consists of all x in the domain of g such that $u = g(x)$ is in the domain of f (Figure 1.8).

Figure 1.8 (a) & (b)

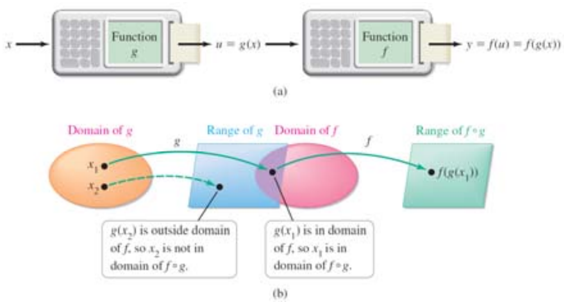


Figure 1.9

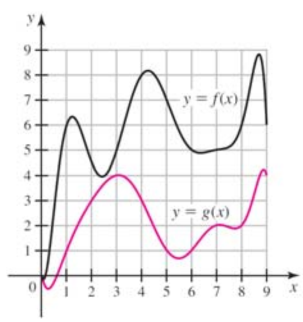


Figure 1.10

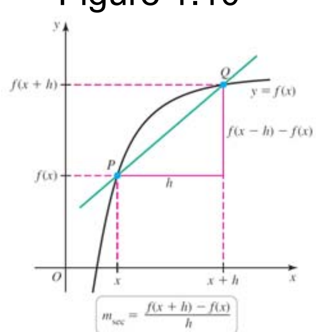


Figure 1.11

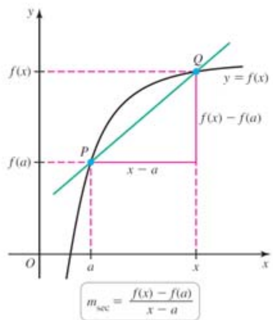
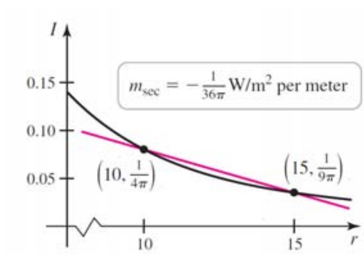


Figure 1.12



DEFINITION Symmetry in Graphs

A graph is **symmetric with respect to the y-axis** if whenever the point (x, y) is on the graph, the point $(-x, y)$ is also on the graph. This property means that the graph is unchanged when reflected about the y-axis (Figure 1.13a).

A graph is **symmetric with respect to the x-axis** if whenever the point (x, y) is on the graph, the point $(x, -y)$ is also on the graph. This property means that the graph is unchanged when reflected about the x-axis (Figure 1.13b).

A graph is **symmetric with respect to the origin** if whenever the point (x, y) is on the graph, the point $(-x, -y)$ is also on the graph (Figure 1.13c). Symmetry about both the x- and y-axes implies symmetry about the origin, but not vice versa.

Figure 1.13 (a)

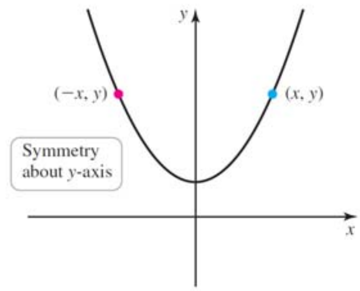


Figure 1.13 (b)

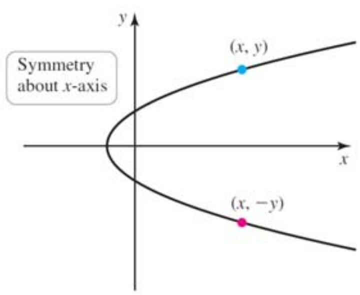
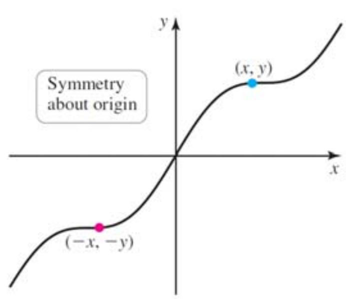


Figure 1.13 (c)



DEFINITION Symmetry in Functions

An **even function** f has the property that $f(-x) = f(x)$, for all x in the domain. The graph of an even function is symmetric about the y-axis. Polynomials consisting of only even powers of the variable (of the form x^{2n}, where n is a nonnegative integer) are even functions.

An **odd function** f has the property that $f(-x) = -f(x)$, for all x in the domain. The graph of an odd function is symmetric about the origin. Polynomials consisting of only odd powers of the variable (of the form x^{2n+1}, where n is a nonnegative integer) are odd functions.

Figure 1.14

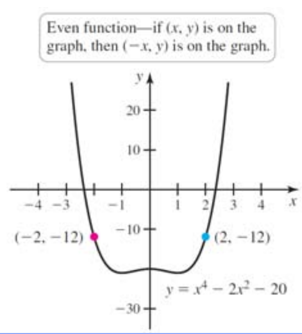


Figure 1.15

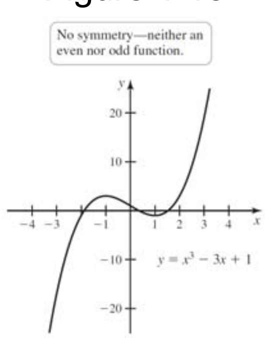
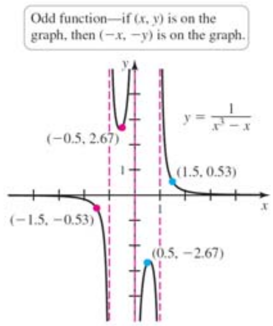


Figure 1.16



1.2

Representing Functions

Figure 1.17

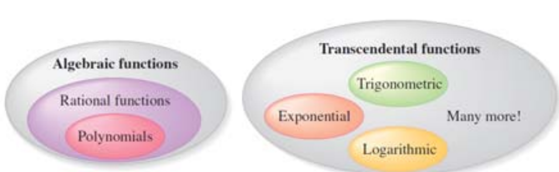
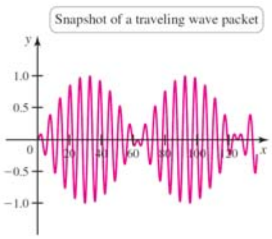


Figure 1.18



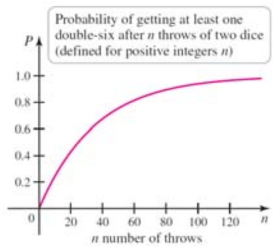


Figure 1.19

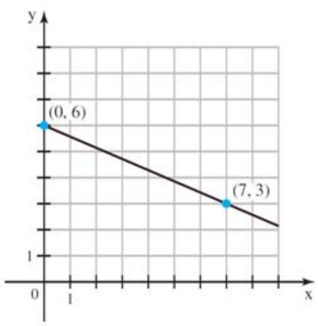


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Figure 1.20

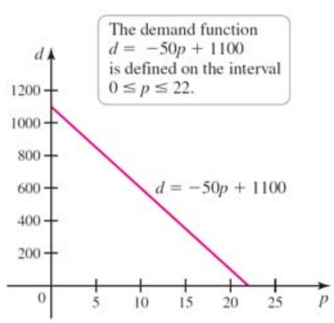
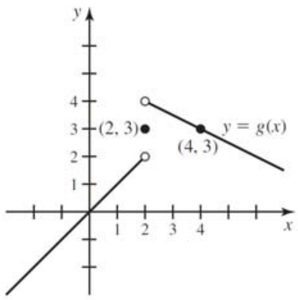


Figure 1.21



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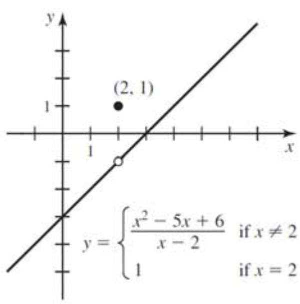

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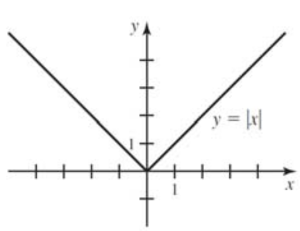

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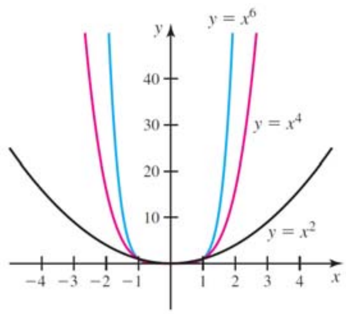

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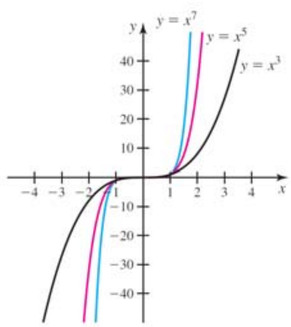

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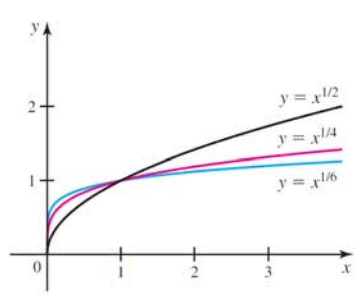


Figure 1.27

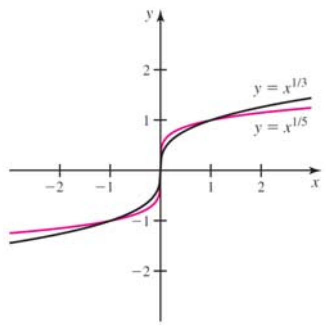


Figure 1.28

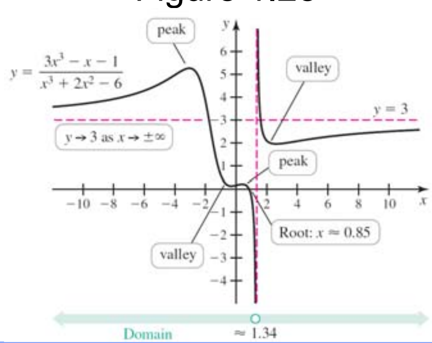


Table 1.1

t (s)	d (cm)
0	0
1	2
2	6
3	14
4	24
5	34
6	44
7	54

Figure 1.29

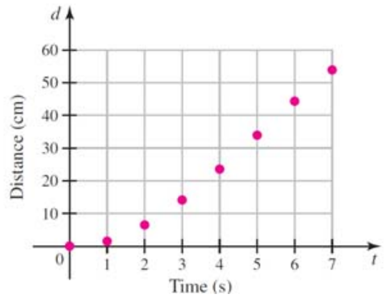


Figure 1.30

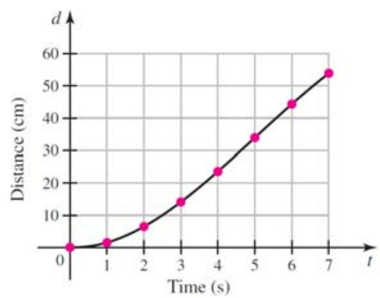


Figure 1.31

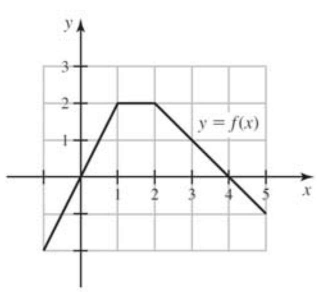


Figure 1.32

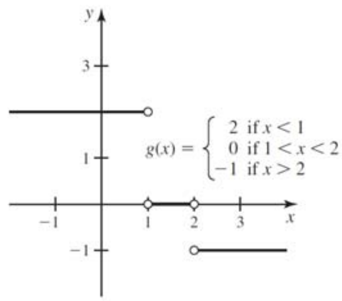


Figure 1.33

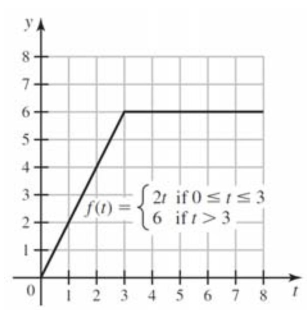


Figure 1.34 (a) & (b)

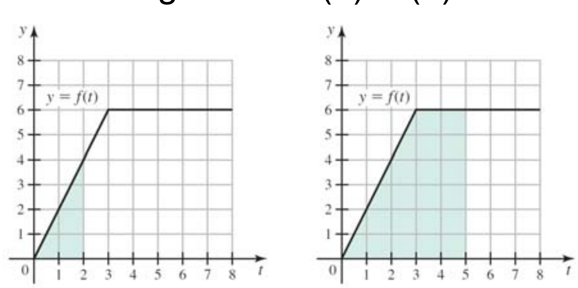


Figure 1.35 (a) & (b)

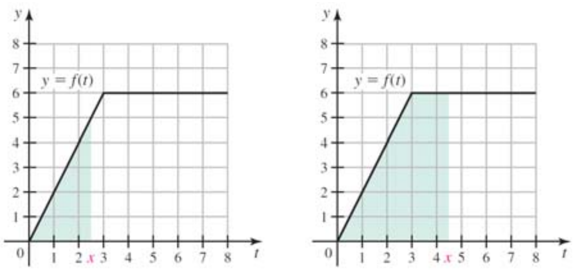


Figure 1.36

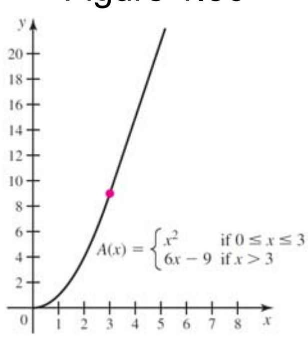


Figure 1.37

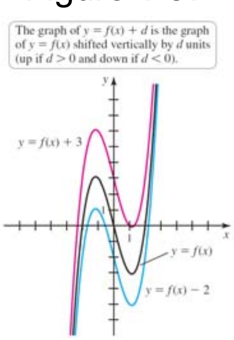


Figure 1.38

The graph of $y = f(x - b)$ is the graph of $y = f(x)$ shifted horizontally by b units (right if $b > 0$ and left if $b < 0$).

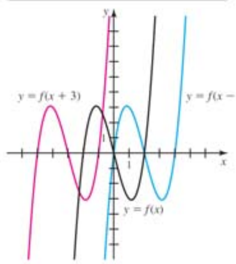


Figure 1.39

For $c > 0$, the graph of $y = cf(x)$ is the graph of $y = f(x)$ scaled vertically by a factor of c (broadened if $0 < c < 1$ and steepened if $c > 1$).

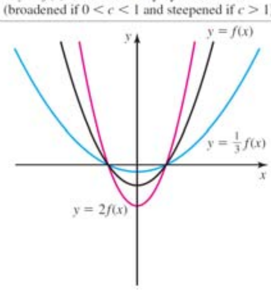


Figure 1.40

For $c < 0$, the graph of $y = cf(x)$ is the graph of $y = f(x)$ scaled vertically by a factor of $|c|$ and reflected across the x-axis (broadened if $-1 < c < 0$ and steepened if $c < -1$).

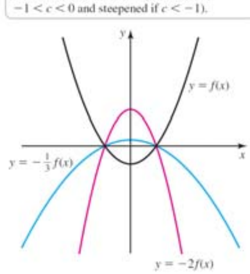


Figure 1.41

For $a > 0$, the graph of $y = f(ax)$ is the graph of $y = f(x)$ scaled horizontally by a factor of a (broadened if $0 < a < 1$ and steepened if $a > 1$).

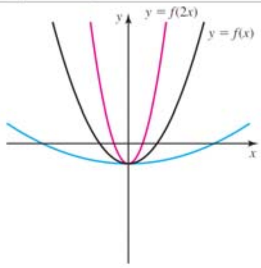


Figure 1.42

For $a < 0$, the graph of $y = f(ax)$ is the graph of $y = f(x)$ scaled horizontally by a factor of $|a|$ and reflected across the y-axis (broadened if $-1 < a < 0$ and steepened if $a < -1$).

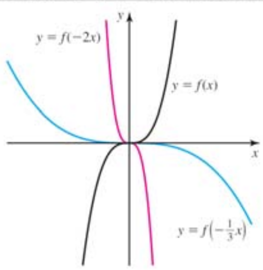


Figure 1.43

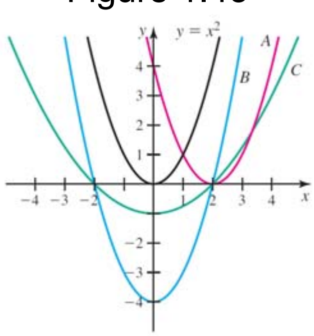
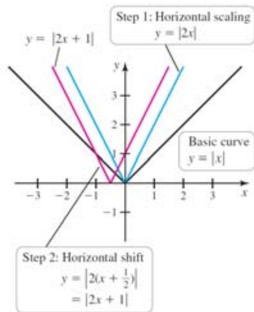


Figure 1.44



SUMMARY Transformations

Given the real numbers a, b, c, and d and the function f, the graph of $y = cf(a(x - b)) + d$ is obtained from the graph of $y = f(x)$ in the following steps.

$$y = f(x) \xrightarrow[\text{by a factor of } |a|]{\text{horizontal scaling}} y = f(ax)$$

$$\xrightarrow[\text{by } b \text{ units}]{\text{horizontal shift}} y = f(a(x - b))$$

$$\xrightarrow[\text{by a factor of } |c|]{\text{vertical scaling}} y = cf(a(x - b))$$

$$\xrightarrow[\text{by } d \text{ units}]{\text{vertical shift}} y = cf(a(x - b)) + d$$

1.3

Trigonometric Functions

Figure 1.45 (a) & (b)

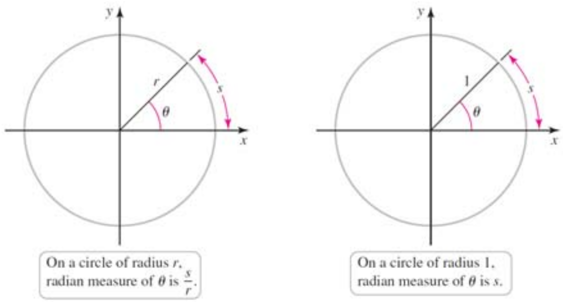
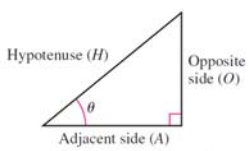


Figure 1.46

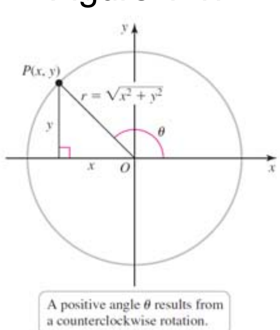


$$\sin\theta = \frac{O}{H} \quad \cos\theta = \frac{A}{H}$$

$$\tan\theta = \frac{O}{A} \quad \cot\theta = \frac{A}{O}$$

$$\sec\theta = \frac{H}{A} \quad \csc\theta = \frac{H}{O}$$

Figure 1.47



DEFINITION **Trigonometric Functions**

Let $P(x, y)$ be a point on a circle of radius r associated with the angle θ. Then

$$\sin\theta = \frac{y}{r} \qquad \cos\theta = \frac{x}{r} \qquad \tan\theta = \frac{y}{x}$$

$$\cot\theta = \frac{x}{y} \qquad \sec\theta = \frac{r}{x} \qquad \csc\theta = \frac{r}{y}$$

Figure 1.48

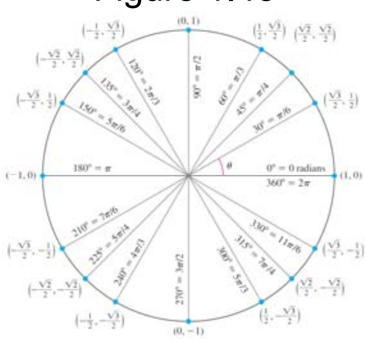


Figure 1.49

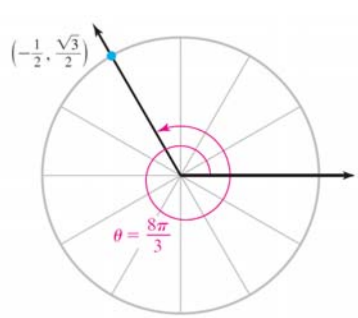
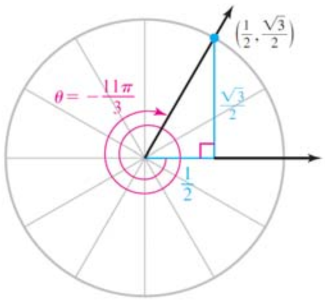


Figure 1.50



Trigonometric Identities

Reciprocal Identities

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cot\theta = \frac{1}{\tan\theta} = \frac{\cos\theta}{\sin\theta}$$

$$\csc\theta = \frac{1}{\sin\theta} \qquad \sec\theta = \frac{1}{\cos\theta}$$

Pythagorean Identities

$$\sin^2\theta + \cos^2\theta = 1 \qquad 1 + \cot^2\theta = \csc^2\theta \qquad \tan^2\theta + 1 = \sec^2\theta$$

Double- and Half-Angle Formulas

$$\sin 2\theta = 2\sin\theta\cos\theta \qquad \cos 2\theta = \cos^2\theta - \sin^2\theta$$

$$\cos^2\theta = \frac{1 + \cos 2\theta}{2} \qquad \sin^2\theta = \frac{1 - \cos 2\theta}{2}$$

Period of Trigonometric Functions

The functions $\sin\theta$, $\cos\theta$, $\sec\theta$, and $\csc\theta$ have a period of 2π:

$$\sin(\theta + 2\pi) = \sin\theta \qquad \cos(\theta + 2\pi) = \cos\theta$$

$$\sec(\theta + 2\pi) = \sec\theta \qquad \csc(\theta + 2\pi) = \csc\theta,$$

for all θ in the domain.

The functions $\tan\theta$ and $\cot\theta$ have a period of π:

$$\tan(\theta + \pi) = \tan\theta \qquad \cot(\theta + \pi) = \cot\theta,$$

for all θ in the domain.

Figure 1.51 (a) & (b)

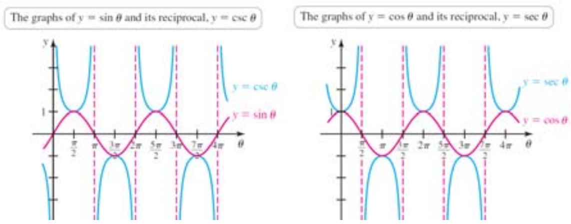


Figure 1.52 (a) & (b)

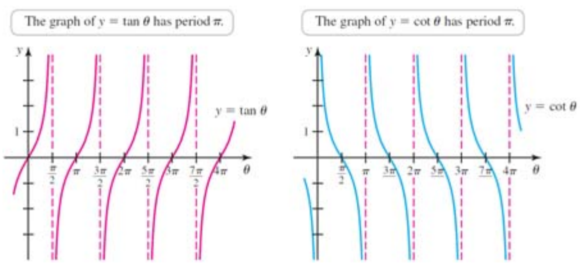


Figure 1.53

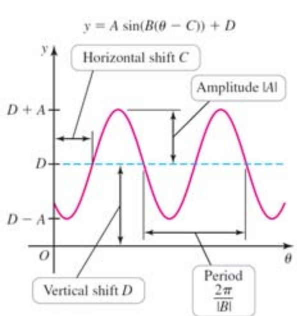
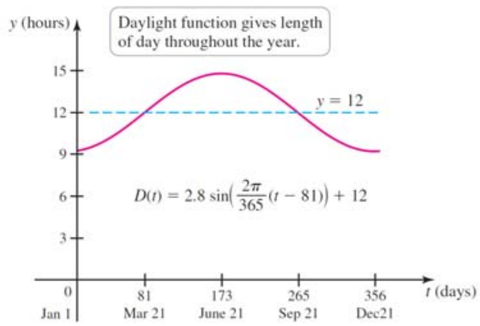


Figure 1.54


y (hours)
Daylight function gives length of day throughout the year.
15
12
9
6
3
y = 12
$D(t) = 2.8 \sin\left(\frac{2\pi}{365}(t - 81)\right) + 12$
0
81
173
265
356
t (days)
Jan 1
Mar 21
June 21
Sep 21
Dec21