# PreCalculus Review for Math 400

1.) Completely factor $6x^3 - 28x^2 - 10x$.

2.) For the function $f(x) = 3x - x^2$, evaluate $f(-2)$.

3.) For the functions $f(x) = \dfrac{1}{x+2}$ and $g(x) = x^2 - 3x$, find and simplify $f(g(x))$. Then, give the domain of $f(g(x))$.

4.) Solve $0 = x^2 - 10x - 10$. Write your solutions in exact, simplified form.

5.) Find the equation of a line that passes through (9, 3) and (4, 5). Put your answer in slope-intercept form.

6.) State the domain of the function $f(x) = \dfrac{\sqrt{x-3}}{x-5}$ using interval notation.

7.) State the range of the function $f(x) = 5 - x^2$ using interval notation.

8.) Rationalize the denominator: $\dfrac{5}{\sqrt{3}}$.

9.) Rationalize the numerator: $\dfrac{5 + \sqrt{3}}{2 - \sqrt{3}}$.

10.) Find the intersection points of the graphs of $y = x^2$ and $y = 2x + 3$.

11.) Find the vertex of $y = x^2 - 6x - 27$.

12.) Find the x and y-intercepts of $y = \dfrac{x-9}{x+2}$.

13.) Simplify $\dfrac{x + \dfrac{1}{x}}{x}$.

14.) Solve $xy - y = 4x + y$ for x.

15.) If $f(x) = 2x + 2$, simplify $\dfrac{f(x+h) - f(x)}{h}$

16.) What is the average rate of change of the function $y = x^2$ on the interval [2, 5]?

17.) Identify each of the following as **a.)** a function or **b.)** a relation, but not a function.
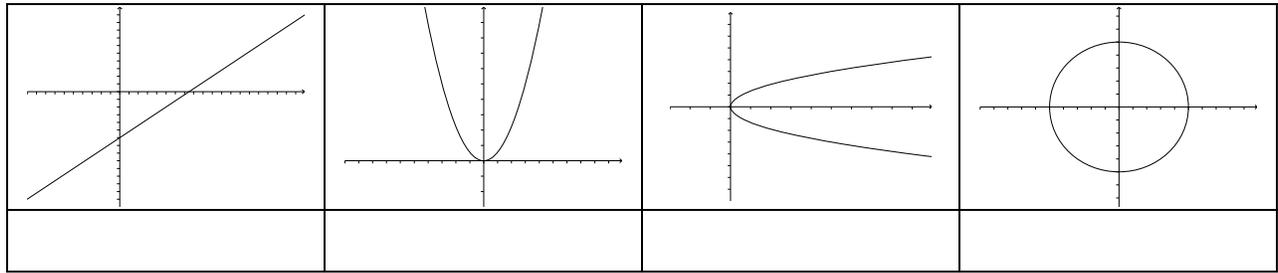
18.) Which of the following is a polynomial function? Circle the correct answer.

a.) $y = \dfrac{x^2}{x+1}$ b.) $y = \sqrt{1-8x}$ c.) $y = \sin x$ d.) $y = 8x^3 - 4x + 3$ e.) $y = e^x + x$

19.) Find the two numbers x and y that satisfy $2x + y = 60$ and have the greatest possible product.
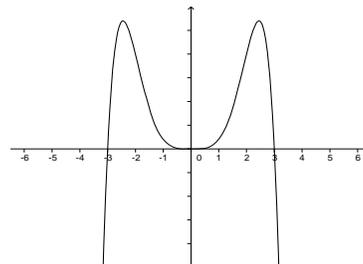
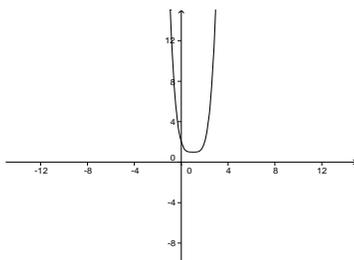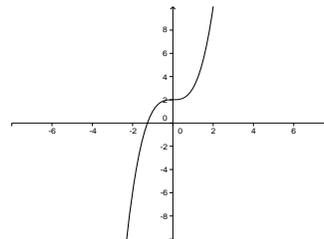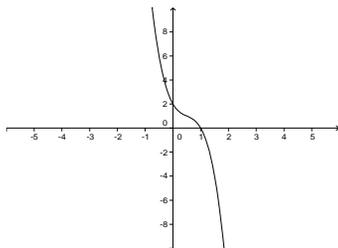20.) If $\dfrac{1}{\sqrt{x^3}} = x^m$, what is m?

21.) Match the formula to the graph by drawing lines.

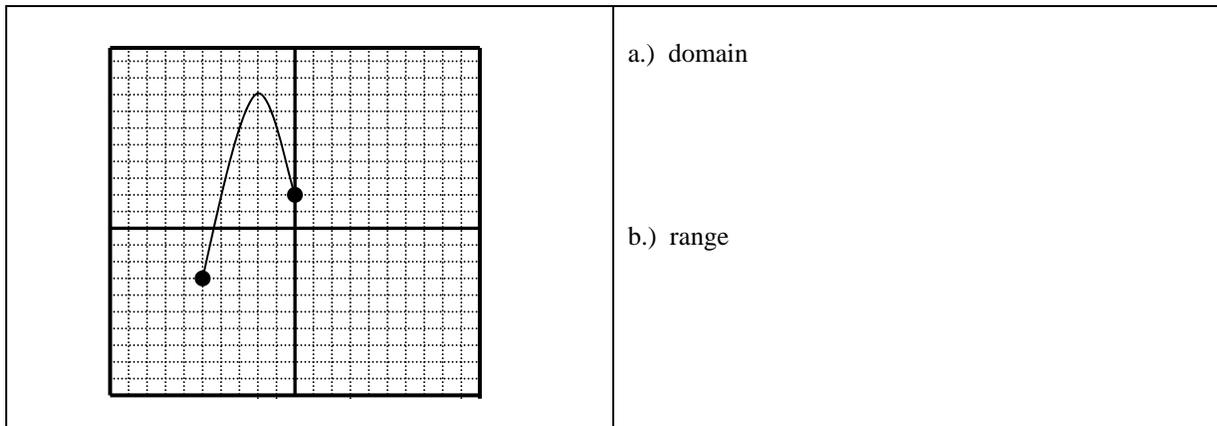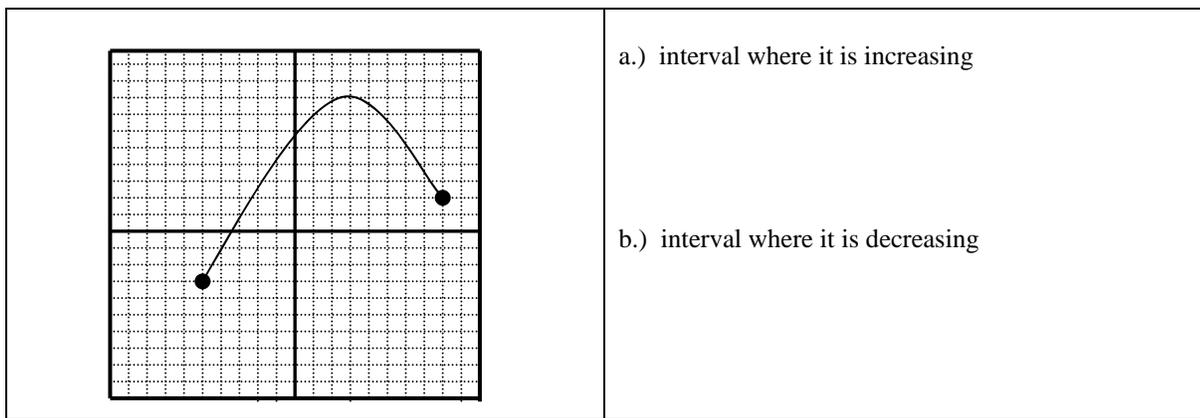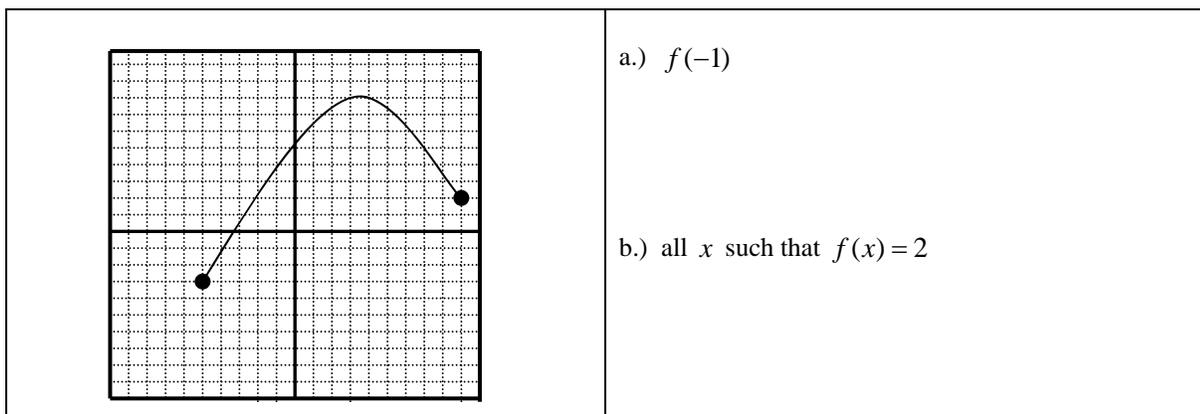$$y = x^4 - 4x^3 + 6x^2 - 4x + 2 \qquad y = -4x^3 + 6x^2 - 4x + 2 \qquad y = x^3 + 2 \qquad y = -x^6 + 9x^4$$

22.) Find the requested features of the function graphed below. Use interval notation.

	a.) domain b.) range

23.) Find the requested features of the function graphed below. Use interval notation.

	a.) interval where it is increasing b.) interval where it is decreasing

24.) Find the requested features of the function f graphed below.

	a.) $f(-1)$ b.) all x such that $f(x) = 2$

25.) Find the requested features of the function f graphed below.

a.) the maximum value of the function

b.) the minimum value of the function
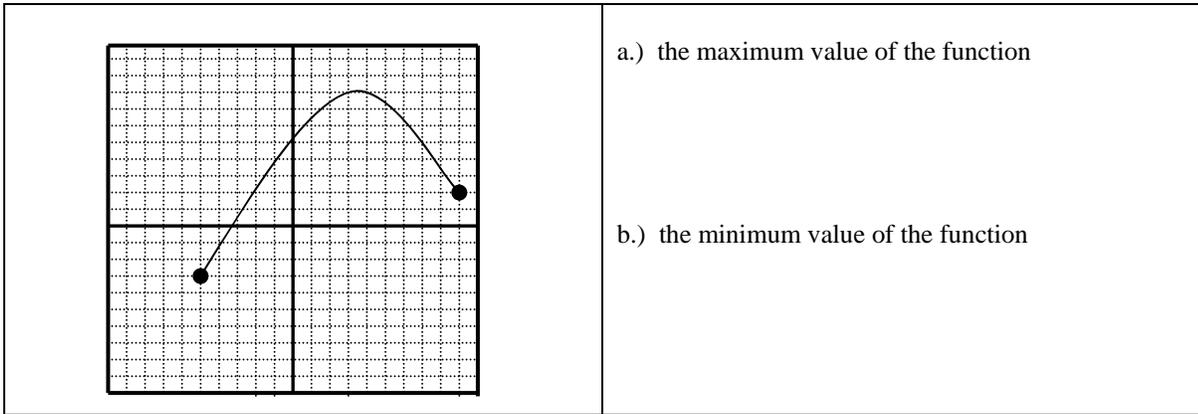
26.) Complete the identities.

$$\sin^2\theta + \cos^2\theta = \boxed{} \qquad \cot^2\theta + 1 = \boxed{} \qquad \boxed{} + 1 = \sec^2\theta$$

27.) Complete the identities.

$$\sin\theta = \dfrac{1}{\boxed{}} \qquad \sec\theta = \dfrac{1}{\boxed{}} \qquad \boxed{} = \dfrac{1}{\tan\theta}$$

28.) Complete the identities.

$$\dfrac{\sin\theta}{\cos\theta} = \boxed{} \qquad \dfrac{\cos\theta}{\sin\theta} = \boxed{}$$
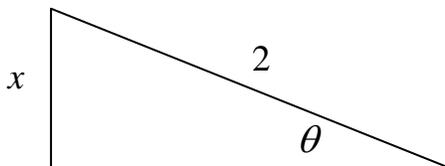
29.) Complete the identities.
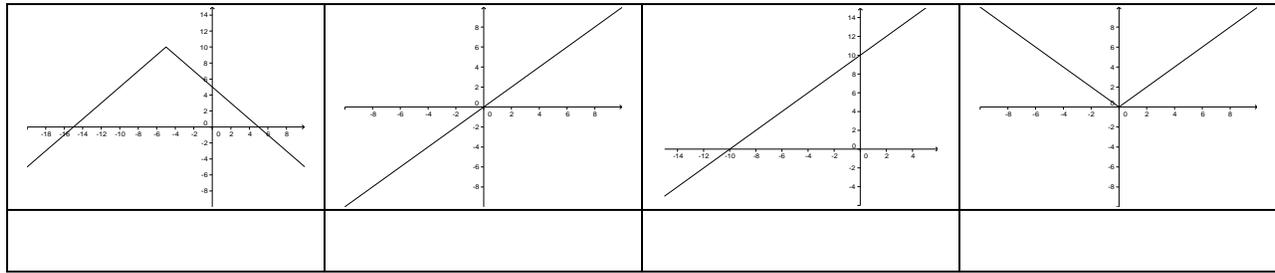
$$\sin(2\theta) = 2\boxed{} \qquad \cos(2\theta) = \boxed{} - 1 \qquad \sin(x + y) = \sin x \cos y + \boxed{}$$

30.) Using the diagram below, write the angle θ as a function of x.

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31.) Label each of the following functions as even, odd, or neither.

32.) Find the sum: $\displaystyle\sum_{x=1}^{3} x^2$

33.) Solve $2\sin(x)+1=0$. Find all solutions in the interval $[0, 2\pi]$.

34.) What is the horizontal asymptote of the function $y = \dfrac{x^2}{x^2+5x-6}$?

35.) What are the vertical asymptotes of the function $y = \dfrac{x^2}{x^2+5x-6}$?

36.) What is the solution set of $|x-4| < \dfrac{1}{2}$?

37.) Write down the formulas for finding the following geometric quantities.

Volume of a Cone   Volume of a Sphere   Surface Area of Sphere

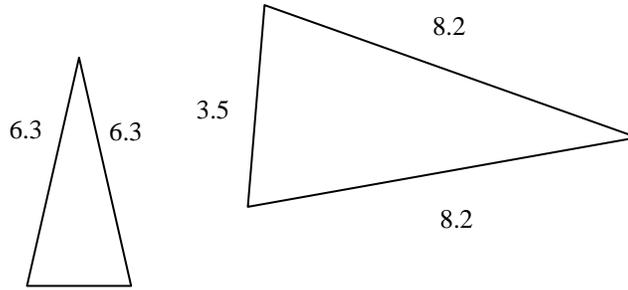Volume of a Cylinder   Lateral Surface Area of a Cylinder   Total Surface Area of a Cylinder

Area of a Trapezoid   Lateral Surface Area of a Cone   Total Surface Area of a Cone

38.) Determine the slant asymptote of $y = \dfrac{x^3+8x-9}{x^2-9}$
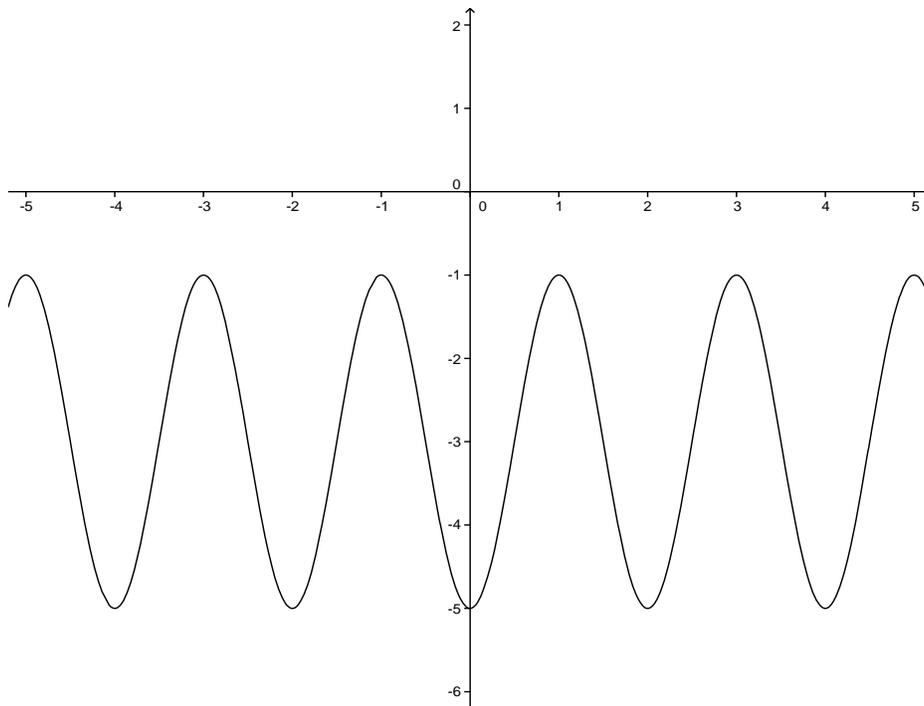
39.) Given that the two triangles below are similar triangles, find the shortest side of the smaller triangle.

8.2

3.5

6.3   6.3

8.2

40.) Solve algebraically: $x^3 + 4x^2 + 4x = -3$

41.) a.) Write an equation for the function below that has the form $y = a\sin(bx + c) + d$

b.) Write an equation for the function below that has the form $y = a\cos(bx + c) + d$

42.) Give the domains and ranges of each of the following functions:

a.) $y = \sin^{-1}(x)$

b.) $y = \cos^{-1}(x)$

c.) $y = \tan^{-1}(x)$

d.) $y = \sec^{-1}(x)$

43.) Find the distance between the points (3, -3) and (-7, -9)

44.) Suppose a and b can be any real numbers. Mark each of the equations below as true or false.

$$|-a| = a \qquad\qquad |b^2| = b^2 \qquad\qquad |a-3| = |3-a|$$

$$|a+1| = |a| + |1| \qquad |a+b| = |a| + |b| \qquad |a-b| = |a| - |b|$$

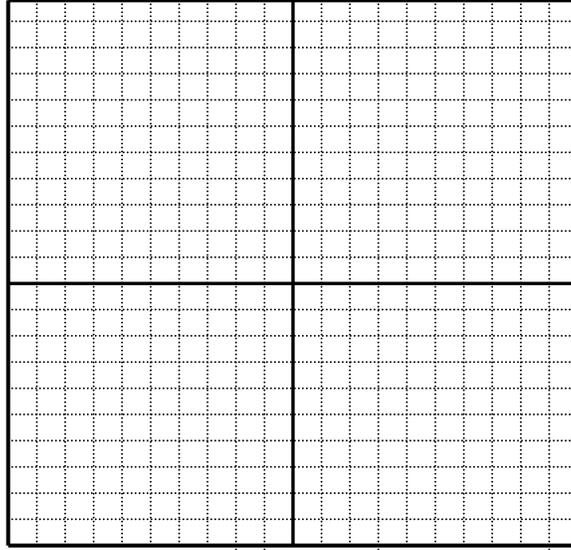$$|ab| = |a| \cdot |b| \qquad \left|\frac{a}{b}\right| = \frac{|a|}{|b|} \qquad |a^2 - b^2| = |a+b| \cdot |a-b|$$

45.) Graph $f(x) = \begin{cases} 2-x, & x < -1 \\ 3, & -1 < x \le 1 \\ x-5, & x > 1 \end{cases}$
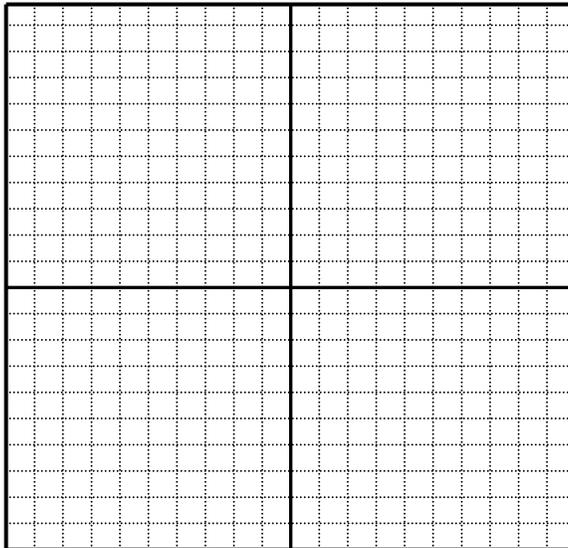
46.) Graph $f(x) = \dfrac{x^2 - 4}{x - 2}$. Show any holes or asymptotes.

47.) Show that $\dfrac{2x(x+1)^{-\frac{1}{2}} - (x+1)^{\frac{1}{2}}}{x^2}$ equals $\dfrac{x-1}{x^2\sqrt{x+1}}$

Key for Review Sheet

1.) $2x(3x+1)(x-5)$

2.) -10

3.) $f(g(x)) = \dfrac{1}{x^2 - 3x + 2}$; $Dom(f \circ g) = (-\infty, 1) \cup (1, 2) \cup (2, \infty)$

4.) $x = 5 \pm \sqrt{35}$

5.) $y = -\dfrac{2}{5}x + \dfrac{33}{5}$

6.) $[3,\ 5) \cup (5,\ \infty)$

7.) $(-\infty,\ 5]$

8.) $\dfrac{5\sqrt{3}}{3}$

9.) $\dfrac{22}{13 - 7\sqrt{3}}$

10.) (3, 9); (-1, 1)

11.) (3, -36)

12.) (9, 0); $\left(0,\ -\dfrac{9}{2}\right)$

13.) $\dfrac{x^2 + 1}{x^2}$

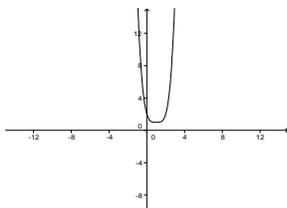14.) $x = \dfrac{2y}{y - 4}$

15.) 2

16.) 7

17.) a, a, b, b

18.) d

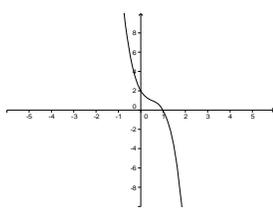19.) x = 15, y = 30

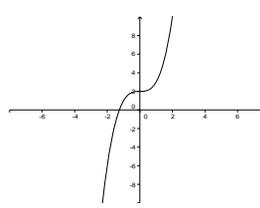20.) $m = -\dfrac{3}{2}$

21.) Match the formula to the graph.

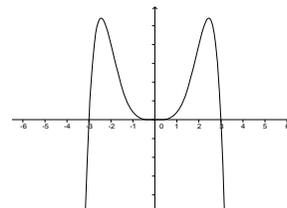$y = x^4 - 4x^3 + 6x^2 - 4x + 2$ $y = -4x^3 + 6x^2 - 4x + 2$ $y = x^3 + 2$ $y = -x^6 + 9x^4$

22.) [-5, 0]; [-3, 8]

23.) [-5, 3]; [3, 8]

24.) 4; -2 and 9

25.) 8; -3

26.) $\sin^2\theta + \cos^2\theta = \boxed{1}$; $\cot^2\theta + 1 = \boxed{\csc^2\theta}$; $\boxed{\tan^2\theta} + 1 = \sec^2\theta$

27.) $\sin\theta = \dfrac{1}{\boxed{\csc\theta}}$; $\sec\theta = \dfrac{1}{\boxed{\cos\theta}}$; $\boxed{\cot\theta} = \dfrac{1}{\tan\theta}$

28.) $\dfrac{\sin\theta}{\cos\theta} = \boxed{\tan\theta}$; $\dfrac{\cos\theta}{\sin\theta} = \boxed{\cot\theta}$

29.) $\sin(2\theta) = 2\boxed{\sin\theta\cos\theta}$; $\cos(2\theta) = \boxed{2\cos^2\theta} - 1$; $\sin(x+y) = \sin x\cos y + \boxed{\sin y\cos x}$

30.) $\theta = \sin^{-1}\left(\dfrac{x}{2}\right)$

31.) neither, odd, neither, even

32.) 14

33.) $\dfrac{7\pi}{6}$, $\dfrac{11\pi}{6}$

34.) $y = 1$

35.) $x = 1$, $x = -6$

36.) $\left(\dfrac{7}{2}, \dfrac{9}{2}\right)$

37.) Write down the formulas for finding the following geometric quantities.

Volume of a Cone	Volume of a Sphere	Surface Area of Sphere
$V = \frac{1}{3}\pi r^2 h$	$V = \frac{4}{3}\pi r^3$	$V = 4\pi r^2$
Volume of a Cylinder	Lateral Surface Area of a Cylinder	Total Surface Area of a Cylinder
$V = \pi r^2 h$	$A = 2\pi rh$	$A = 2\pi rh + 2\pi r^2$
Area of a Trapezoid	Lateral Surface Area of a Cone	Total Surface Area of a Cone
$A = \frac{1}{2}h(b_1 + b_2)$	$A = \pi r\sqrt{r^2 + h^2}$	$A = \pi r\sqrt{r^2 + h^2} + \pi r^2$

38.) $y = x$

39.) ≈ 2.7

40.) -3, $-\dfrac{1}{2} \pm \dfrac{\sqrt{3}}{2}i$

41.) a.) $y = 2\sin\left(\pi x - \dfrac{\pi}{2}\right) - 3$ (other answers possible)

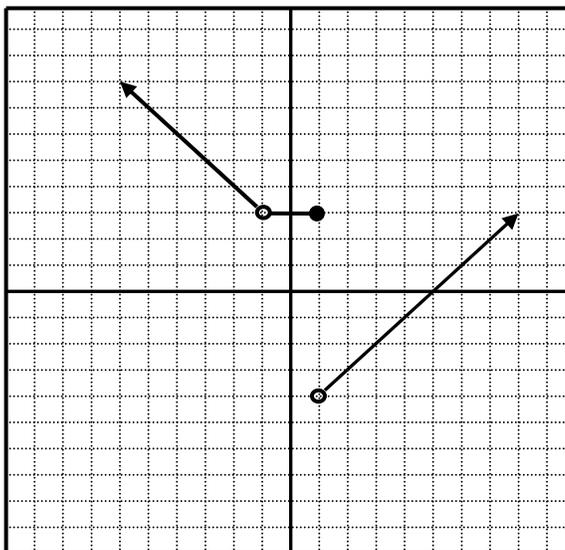   b.) $y = -2\cos(\pi x) - 3$ (other answers possible)

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42.)

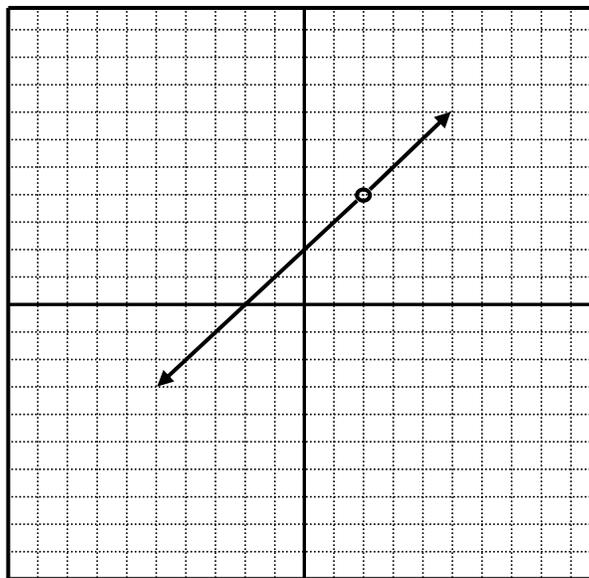| Function	Domain	Range
$y = \sin^{-1}(x)$	$[-1,\ 1]$	$\left[-\dfrac{\pi}{2},\ \dfrac{\pi}{2}\right]$
$y = \cos^{-1}(x)$	$[-1,\ 1]$	$[0,\ \pi]$
$y = \tan^{-1}(x)$	$(-\infty,\ \infty)$	$\left[-\dfrac{\pi}{2},\ \dfrac{\pi}{2}\right]$
$y = \sec^{-1}(x)$	$(-\infty,\ -1]\cup[1,\ \infty)$	$\left[0,\ \dfrac{\pi}{2}\right)\cup\left(\dfrac{\pi}{2},\ \pi\right]$

43.) $2\sqrt{34}$ or ≈ 11.66

44.)

$|-a| = a\ \rightarrow$ False $\qquad\qquad |b^2| = b^2\ \rightarrow$ True $\qquad\qquad |a-3| = |3-a|\ \rightarrow$ True

$|a+1| = |a|+|1|\ \rightarrow$ False $\quad |a+b| = |a|+|b|\ \rightarrow$ False $\qquad |a-b| = |a|-|b|\ \rightarrow$ False

$|ab| = |a|\cdot|b|\ \rightarrow$ True $\qquad\qquad \left|\dfrac{a}{b}\right| = \dfrac{|a|}{|b|}\ \rightarrow$ True $\qquad |a^2 - b^2| = |a+b|\cdot|a-b|\ \rightarrow$ True

45.) Graph $f(x) = \begin{cases} 2-x, & x < -1 \\ 3, & -1 < x \leq 1 \\ x-5, & x > 1 \end{cases}$

46.) Graph $f(x) = \dfrac{x^2 - 4}{x - 2}$. Show any holes or asymptotes.

47.) Show that $\dfrac{2x(x+1)^{-\frac{1}{2}} - (x+1)^{\frac{1}{2}}}{x^2}$ equals $\dfrac{x-1}{x^2\sqrt{x+1}}$

    a) Factor the numerator: $\dfrac{(x+1)^{-\frac{1}{2}}\left[2x - (x+1)^1\right]}{x^2}$

    b) Simplify inside the brackets: $\dfrac{(x+1)^{-\frac{1}{2}}[x-1]}{x^2}$

    c) Bring $(x+1)^{-\frac{1}{2}}$ to the denominator and drop the negative: $\dfrac{[x-1]}{(x+1)^{\frac{1}{2}}\,x^2}$

    d) Rewrite the fractional exponent as a radical: $\dfrac{x-1}{x^2\sqrt{x+1}}$