How to find the equation of tangent line at a given point by using derivative

Example 1: Given $y = f(x) = 2x^3 - 4x^2 + 6x - 3$ find the equation of tangent line at $x = 2$

Step 1 $x = 2$ $\quad y = f(2) = 2(2)^3 - 4(2)^2 + 6(2) - 3 = 9$ so the point will be $(2,9)$

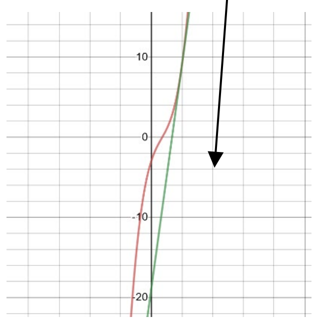
Step 2 Now to find general slope of the tangent line, we need to find $y' = f'(x) = 6x^2 - 8x + 6$

Step 3 now at $x = 2$ $\quad m = f'(2) = 6(2)^2 - 8(2) + 6 = 14$

Step 4 So we use point slope formula to find the equation of tangent line $y - y_1 = m(x - x_1)$

$y - 9 = 14(x - 2)$ then **final answer** is $y = 14x - 19$

Check: graph both $y = f(x)$ and tangent equation in Desmos to see if it is correctly tangent to $f(x)$ at $x = 2$



Example 2: Given $y = f(x) = 2\sin 2x - 3\cos x$ find the equation of tangent line at $x = \pi$

Step 1 $x = \pi$ $\quad y = f(\pi) = 2\sin(2\pi) - 3\cos(\pi) = 0 - 3(-1) = 3$ so the point will be $(\pi, 3)$

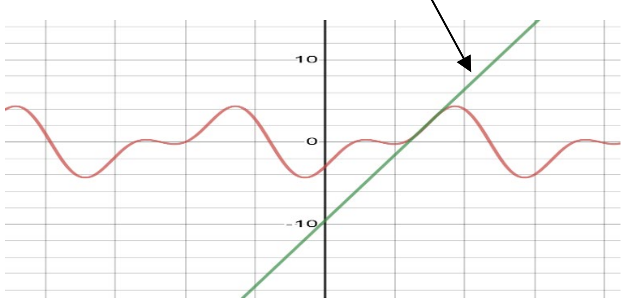
Step 2 Now to find general slope of the tangent line, we need to find $y' = f'(x) = 2(2\cos(2x)) - 3(-\sin x)$

Step 3 now at $x = 2$ $\quad m = f'(\pi) = f'(x) = 2(2\cos(2\pi)) - 3(-\sin \pi) = 4$

Step 4 So we use point slope formula to find the equation of tangent line $y - y_1 = m(x - x_1)$

$y - 3 = 4(x - \pi)$ then **final answer** is $y = 4x - 4\pi + 3$

Check: graph both $y = f(x)$ and tangent equation in Desmos to see if it is correctly tangent to $f(x)$ at $x = \pi$



Example 3: Given the ellipse $\frac{x^2}{4}+\frac{y^2}{9}=1$ or $9x^2+4y^2=36$ find the equation of tangent line at $x=1$

Step 1 $x=1 \quad 9(1)^2+4y^2=36 \quad 4y^2=27 \quad y=\pm\sqrt{\frac{27}{4}}=\pm 2.6$ so the point will be $(1,2.6)$ and $(1,-2.6)$

Step 2 Now to find general slope of the tangent line, we need to find derivative by using implicit differentiation

$$18x+8yy'=0 \qquad 9x+4yy'=0 \qquad m=y'=-\frac{9x}{4y}$$

Step 3 Now because we have two points then we will be having two slopes

$$m_1=y'=-\frac{9(1)}{4(2.6)}=-.865 \quad \text{and} \quad m_2=y'=-\frac{9(1)}{4(-2.6)}=.865$$

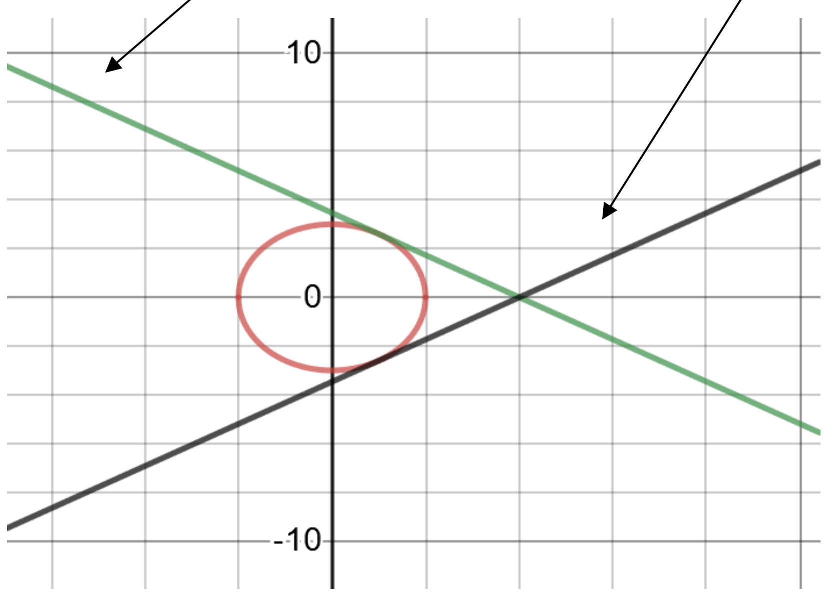
Step 4 So we use point slope formula to find the equation of tangent line $y-y_1=m(x-x_1)$

At $(1,2.6)$ and $m_1=-.865 \quad y-2.6=-.865(x-1) \quad y=-.865x+3.465$

At $(1,-2.6)$ and $m_1=.865 \quad y+2.6=.865(x-1) \quad y=.865x-3.465$

then **final answer** is $y=4x-4\pi+3$

Check: graph both $y=f(x)$ and tangent equation in Desmos to see if it is correctly tangent to $f(x)$ at $(1,2.6)$ and $(1,-2.6)$



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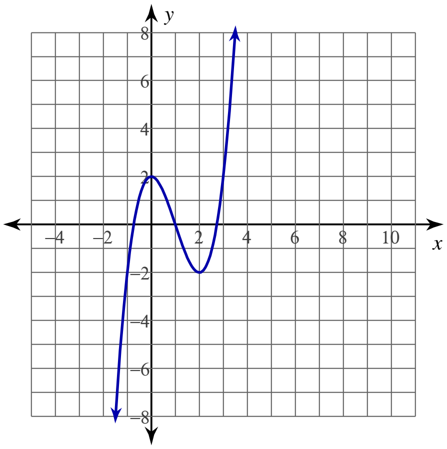
Tangent Lines

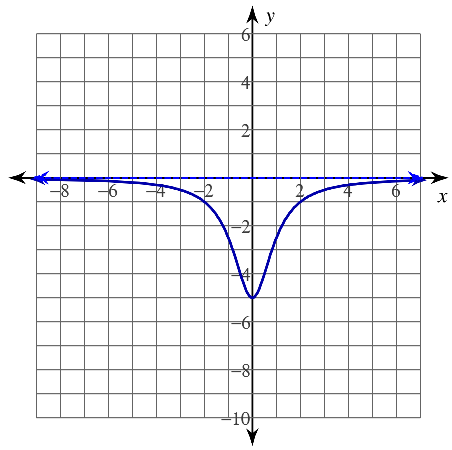
Date________________ Period____

For each problem, find the equation of the line tangent to the function at the given point. Your answer should be in slope-intercept form.

1) $y = x^3 - 3x^2 + 2$ at $(3, 2)$



2) $y = -\dfrac{5}{x^2 + 1}$ at $\left(-1, -\dfrac{5}{2}\right)$



3) $y = x^3 - 2x^2 + 2$ at $(2, 2)$

4) $y = -\dfrac{3}{x^2 - 25}$ at $\left(-4, \dfrac{1}{3}\right)$

5) $y = -\dfrac{3}{x^2 - 4}$ at $(1, 1)$

6) $y = (5x + 5)^{\frac{1}{2}}$ at $(4, 5)$

7) $y = \ln(-x)$ at $(-2, \ln 2)$

8) $y = -2\tan(x)$ at $(-\pi, 0)$

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Tangent Lines

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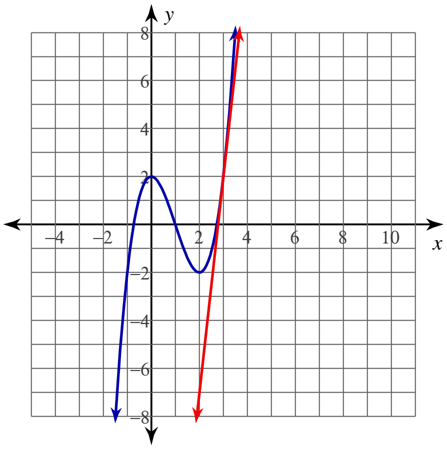
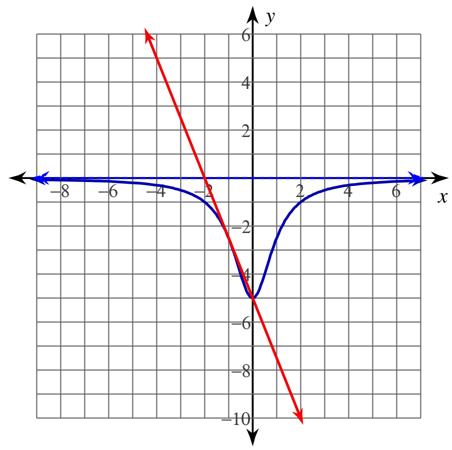
1) $y = x^3 - 3x^2 + 2$ at $(3, 2)$



$y = 9x - 25$

2) $y = -\dfrac{5}{x^2 + 1}$ at $\left(-1, -\dfrac{5}{2}\right)$



$y = -\dfrac{5}{2}x - 5$

3) $y = x^3 - 2x^2 + 2$ at $(2, 2)$

$y = 4x - 6$

4) $y = -\dfrac{3}{x^2 - 25}$ at $\left(-4, \dfrac{1}{3}\right)$

$y = -\dfrac{8}{27}x - \dfrac{23}{27}$

5) $y = -\dfrac{3}{x^2 - 4}$ at $(1, 1)$

$y = \dfrac{2}{3}x + \dfrac{1}{3}$

6) $y = (5x + 5)^{\frac{1}{2}}$ at $(4, 5)$

$y = \dfrac{1}{2}x + 3$

7) $y = \ln(-x)$ at $(-2, \ln 2)$

$y = -\dfrac{1}{2}x + \ln 2 - 1$

8) $y = -2\tan(x)$ at $(-\pi, 0)$

$y = -2x - 2\pi$

1. Find the equation of the tangent line to the graph of the given function at the given point:
$f(x) = x - 3x^2$; $P(-2, -14)$

2. Find the equation of the tangent line to the graph of the given function at the given point:
$f(x) = -1 + 2x + 3x^2$; $P(0, -1)$

3. Find the equation of the tangent line to the graph of the given function at the given point:
$f(x) = -2 - x^2$; $P(2, -6)$

4. Find the equation of the tangent line to the graph of the given function at the given point:
$f(x) = x - x^2$; $P(0, 0)$

5. Find the equation of the tangent line to the graph of the given function at the given point:
$f(x) = -3 + 2x + x^2$; $P(3, 12)$

6. Find the equation of the tangent line to the graph of the given function at the given point:
$f(x) = -3 + 2x - 2x^2$; $P(3, -15)$

7. Find the equation of the tangent line to the graph of the given function at the given point:
$f(x) = 1 - 2x + x^2$; $P(0, 1)$

8. Find the equation of the tangent line to the graph of the given function at the given point:
$f(x) = 3 + 2x - 3x^2$; $P(1, 2)$

9. Find the equation of the tangent line to the graph of the given function at the given point:
$f(x) = 3 - 2x^2$; $P(-1, 1)$

10. Find the equation of the tangent line to the graph of the given function at the given point:
$f(x) = 2 + x + 3x^2$; $P(0, 2)$

Answers:

1. $y = 13x + 12$ 2. $y = 2x - 1$ 3. $y = -4x + 2$ 4. $y = 1x0$ 5. $y = 8x - 12$ 6. $y = -10x + 15$ 7. $y = -2x + 1$ 8. $y = -4x + 6$ 9. $y = 4x + 5$ 10. $y = 1x + 2$

Solutions:

1. $f'(x) = \dfrac{\mathrm{d}}{\mathrm{d}x}(x - 3x^2) = 1 - 6x$ ◀ Find the first derivative of the function.

$m = f'(-2) = 1 - 6(-2) = 13$ ◀ Find the slope of the tangent line at the given point P.

$y - (-14) = 13[x - (-2)]$ ◀ Use the Point-Slope formula: $y - y_1 = m(x - x_1)$ ▶ Then simplify:

$y = 13x + 12$

2. $f'(x) = \dfrac{\mathrm{d}}{\mathrm{d}x}(-1 + 2x + 3x^2) = 2 + 6x$ ◀ Find the first derivative of the function.

$m = f'(0) = 2 + 6(0) = 2$ ◀ Find the slope of the tangent line at the given point P.

$y - (-1) = 2[x - (0)]$ ◀ Use the Point-Slope formula: $y - y_1 = m(x - x_1)$ ▶ Then simplify:

$y = 2x - 1$

3. $f'(x) = \dfrac{\mathrm{d}}{\mathrm{d}x}(-2 - x^2) = -2x$ ◀ Find the first derivative of the function.

$m = f'(2) = -2(2) = -4$ ◀ Find the slope of the tangent line at the given point P.

$y - (-6) = -4[x - (2)]$ ◀ Use the Point-Slope formula: $y - y_1 = m(x - x_1)$ ▶ Then simplify:

$y = -4x + 2$

4. $f'(x) = \dfrac{\mathrm{d}}{\mathrm{d}x}(x - x^2) = 1 - 2x$ ◀ Find the first derivative of the function.

$m = f'(0) = 1 - 2(0) = 1$ ◀ Find the slope of the tangent line at the given point P.

$y - (0) = 1[x - (0)]$ ◀ Use the Point-Slope formula: $y - y_1 = m(x - x_1)$ ▶ Then simplify:

$y = x0$

5. $f'(x) = \dfrac{\mathrm{d}}{\mathrm{d}x}(-3 + 2x + x^2) = 2 + 2x$ ◀ Find the first derivative of the function.

$m = f'(3) = 2 + 2(3) = 8$ ◀ Find the slope of the tangent line at the given point P.

$y - (12) = 8[x - (3)]$ ◀ Use the Point-Slope formula: $y - y_1 = m(x - x_1)$ ▶ Then simplify:

$y = 8x - 12$

6. $f'(x) = \dfrac{\mathrm{d}}{\mathrm{d}x}(-3 + 2x - 2x^2) = 2 - 4x$ ◀ Find the first derivative of the function.

$m = f'(3) = 2 - 4(3) = -10$ ◀ Find the slope of the tangent line at the given point P.

$y - (-15) = -10[x - (3)]$ ◀ Use the Point-Slope formula: $y - y_1 = m(x - x_1)$ ▶ Then simplify:

$y = -10x + 15$

7. $f'(x) = \dfrac{\mathrm{d}}{\mathrm{d}x}(1 - 2x + x^2) = -2 + 2x$ ◀ Find the first derivative of the function.

$m = f'(0) = -2 + 2(0) = -2$ ◀ Find the slope of the tangent line at the given point P.

$y - (1) = -2[x - (0)]$ ◀ Use the Point-Slope formula: $y - y_1 = m(x - x_1)$ ▶ Then simplify:

$y = -2x + 1$

8. $f'(x) = \dfrac{\mathrm{d}}{\mathrm{d}x}(3 + 2x - 3x^2) = 2 - 6x$ ◀ Find the first derivative of the function.

$m = f'(1) = 2 - 6(1) = -4$ ◀ Find the slope of the tangent line at the given point P.

$y - (2) = -4[x - (1)]$ ◀ Use the Point-Slope formula: $y - y_1 = m(x - x_1)$ ▶ Then simplify:

$y = -4x + 6$

9. $f'(x) = \dfrac{\mathrm{d}}{\mathrm{d}x}(3 - 2x^2) = -4x$ ◄ Find the first derivative of the function.

$m = f'(-1) = -4(-1) = 4$ ◄ Find the slope of the tangent line at the given point P.

$y - (1) = 4[x - (-1)]$ ◄ Use the Point-Slope formula: $y - y_1 = m(x - x_1)$ ► Then simplify:

$y = 4x + 5$

10. $f'(x) = \dfrac{\mathrm{d}}{\mathrm{d}x}(2 + x + 3x^2) = 1 + 6x$ ◄ Find the first derivative of the function.

$m = f'(0) = 1 + 6(0) = 1$ ◄ Find the slope of the tangent line at the given point P.

$y - (2) = 1[x - (0)]$ ◄ Use the Point-Slope formula: $y - y_1 = m(x - x_1)$ ► Then simplify:

$y = x + 2$

