AMERICAN RIVER COLLEGE
MATHEMATICS DEPARTMENT
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MATH 100: BEGINNING ALGEBRA
FINAL REVIEW
SIMPLIFYING

5.1 THE PRODUCT RULE AND POWER RULES FOR EXPONENTS

For any integers m and n,

Product Rule $a^m \cdot a^n = a^{m+n}$

Power Rules (a) $(a^m)^n = a^{mn}$

(b) $(ab)^m = a^m b^m$

(c) $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$ $(b \neq 0)$.

Perform the operations by using rules for exponents.

$$2^4 \cdot 2^5 = 2^{4+5} = 2^9$$

$$(3^4)^2 = 3^{4 \cdot 2} = 3^8$$

$$(6a)^5 = 6^5 a^5$$

$$\left(\frac{2}{3}\right)^4 = \frac{2^4}{3^4}$$

5.2 INTEGER EXPONENTS AND THE QUOTIENT RULE

If $a \neq 0$, then for integers m and n,

Zero Exponent $a^0 = 1$

Negative Exponent $a^{-n} = \frac{1}{a^n}$

Quotient Rule $\frac{a^m}{a^n} = a^{m-n}$

Negative-to-Positive Rules

$$\frac{a^{-m}}{b^{-n}} = \frac{b^n}{a^m} \quad (b \neq 0)$$

$$\left(\frac{a}{b}\right)^{-m} = \left(\frac{b}{a}\right)^m \quad (b \neq 0).$$

Simplify by using the rules for exponents.

$$15^0 = 1$$

$$5^{-2} = \frac{1}{5^2} = \frac{1}{25}$$

$$\frac{4^8}{4^3} = 4^{8-3} = 4^5$$

$$\frac{4^{-2}}{3^{-5}} = \frac{3^5}{4^2}$$

$$\left(\frac{6}{5}\right)^{-3} = \left(\frac{5}{6}\right)^3$$

1) Simplify.

$$\left(\frac{x^{-3}\left(yz^2\right)^2}{x^2y^3z^3}\right)^{-2}$$

2) Simplify.

$$\left(-3x^4\right)\left(7x^{-5}\right)$$

5.3 AN APPLICATION OF EXPONENTS: SCIENTIFIC NOTATION

To write a number in scientific notation (as $a \times 10^n$, where $1 \leq |a| < 10$), move the decimal point to follow the first nonzero digit. If moving the decimal point makes the number smaller, n is positive. If it makes the number larger, n is negative. If the decimal point is not moved, n is 0.

Write in scientific notation.

$$247 = 2.47 \times 10^2$$
$$0.0051 = 5.1 \times 10^{-3}$$
$$4.8 = 4.8 \times 10^0$$

Write without exponents.

$$3.25 \times 10^5 = 325{,}000$$
$$8.44 \times 10^{-6} = 0.00000844$$

3) Convert to scientific notation:

-0.00000062

4) Convert to a decimal:

-3.2×10^7

5.4 ADDING AND SUBTRACTING POLYNOMIALS; GRAPHING SIMPLE POLYNOMIALS

Adding Polynomials
Add like terms.

Add.

$$\begin{array}{r} 2x^2 + 5x - 3 \\ \underline{5x^2 - 2x + 7} \\ 7x^2 + 3x + 4 \end{array}$$

Subtracting Polynomials
Change the signs of the terms in the second polynomial and add the second polynomial to the first.

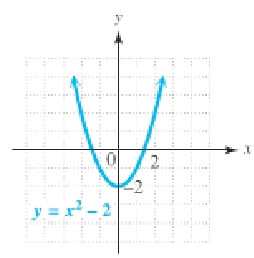
Subtract.

$$\begin{aligned} &(2x^2 + 5x - 3) - (5x^2 - 2x + 7) \\ &= (2x^2 + 5x - 3) + (-5x^2 + 2x - 7) \\ &= -3x^2 + 7x - 10 \end{aligned}$$

Graphing Simple Polynomials
To graph a simple polynomial equation such as $y = x^2 - 2$, plot points near the vertex. (In this chapter, all parabolas have a vertex on the x-axis or the y-axis.)

Graph $y = x^2 - 2$.

x	y
−2	2
−1	−1
0	−2
1	−1
2	2



5) Subtract:

$$\begin{array}{l} 3x^3 - 5x^2 + 3x - 7 \\ 7x^3 - 5x^2 - 5x - 3 \end{array}$$

5.5 MULTIPLYING POLYNOMIALS

General Method for Multiplying Polynomials
Multiply each term of the first polynomial by each term of the second polynomial. Then add like terms.

Multiply.

$$\begin{array}{r} 3x^3 - 4x^2 + 2x - 7 \\ 4x + 3 \\ \hline 9x^3 - 12x^2 + 6x - 21 \\ 12x^4 - 16x^3 + 8x^2 - 28x \\ \hline 12x^4 - 7x^3 - 4x^2 - 22x - 21 \end{array}$$

FOIL Method for Multiplying Binomials

Step 1 Multiply the two **F**irst terms to get the first term of the answer.

Step 2 Find the **O**uter product and the **I**nner product, and mentally add them, when possible, to get the middle term of the answer.

Step 3 Multiply the two **L**ast terms to get the last term of the answer.

Add the terms found in Steps 1–3.

Multiply. $(2x + 3)(5x - 4)$

$2x(5x) = 10x^2$

$2x(-4) + 3(5x) = 7x$

$3(-4) = -12$

The product of $(2x + 3)$ and $(5x - 4)$ is $10x^2 + 7x - 12$.

6) Multiply:

$(x + 3y)(x - 5y)$

7) Multiply:

$$\begin{array}{r} x^3 - 2x^2 + 3x - 7 \\ x - 3 \\ \hline \end{array}$$

5.6 SPECIAL PRODUCTS

Square of a Binomial

$(x + y)^2 = x^2 + 2xy + y^2$

$(x - y)^2 = x^2 - 2xy + y^2$

Product of the Sum and Difference of Two Terms

$(x + y)(x - y) = x^2 - y^2$

Multiply.

$(3x + 1)^2 = 9x^2 + 6x + 1$

$(2m - 5n)^2 = 4m^2 - 20mn + 25n^2$

$(4a + 3)(4a - 3) = 16a^2 - 9$

8) Multiply:

$(x + 3)^2$

9) Multiply:

$(2x - 5y)^2$

5.7 DIVIDING POLYNOMIALS

Dividing a Polynomial by a Monomial
Divide each term of the polynomial by the monomial.

$$\frac{a+b}{c} = \frac{a}{c} + \frac{b}{c}$$

Dividing a Polynomial by a Polynomial
Use "long division."

Divide.

$$\frac{4x^3 - 2x^2 + 6x - 9}{2x} = 2x^2 - x + 3 - \frac{9}{2x}$$

$$\begin{array}{r} 2x - 5 \\ 3x + 4\overline{)6x^2 - 7x - 21} \\ \underline{6x^2 + 8x} \\ -15x - 21 \\ \underline{-15x - 20} \\ -1 \end{array} \leftarrow \text{Remainder}$$

The final answer is $2x - 5 + \frac{-1}{3x+4}$.

10) Divide:

$$\left(12x^3 - 6x + 3\right) \div (3x)$$

11) Divide:

$$\left(x^3 - 8x^2 + 19x - 10\right) \div (x - 3)$$

6.1 THE GREATEST COMMON FACTOR; FACTORING BY GROUPING

Finding the Greatest Common Factor (GCF)

1. Include the largest numerical factor of every term.
2. Include each variable that is a factor of every term raised to the least exponent that appears in a term.

Find the greatest common factor of

$$4x^2y, \quad -6x^2y^3, \quad 2xy^2.$$

$$4x^2y = 2^2 \cdot x^2 \cdot y$$

$$-6x^2y^3 = -1 \cdot 2 \cdot 3 \cdot x^2 \cdot y^3$$

$$2xy^2 = 2 \cdot x \cdot y^2$$

The greatest common factor is $2xy$.

Factoring by Grouping

Step 1 Group the terms.

Step 2 Factor out the greatest common factor in each group.

Step 3 Factor out a common binomial factor from the result of Step 2.

Step 4 If necessary, try a different grouping.

Factor by grouping.

$$3x^2 + 5x - 24xy - 40y = (3x^2 + 5x) + (-24xy - 40y)$$

$$= x(3x + 5) - 8y(3x + 5)$$

$$= (3x + 5)(x - 8y)$$

(continued)

12) Factor: $12x^3y - 6y^3$

13) Factor: $3x^2 - 7x - 3xy + 7y$

6.2 FACTORING TRINOMIALS

To factor $x^2 + bx + c$, find m and n such that $mn = c$ and $m + n = b$.

$$mn = c$$

$$x^2 + bx + c$$

$$m + n = b$$

Then $\quad x^2 + bx + c = (x + m)(x + n)$.

Check by multiplying.

Factor $x^2 + 6x + 8$.

$$mn = 8$$

$$x^2 + 6x + 8$$

$$m + n = 6$$

$m = 2$ and $n = 4$

$$x^2 + 6x + 8 = (x + 2)(x + 4)$$

Check: $(x + 2)(x + 4) = x^2 + 4x + 2x + 8$

$$= x^2 + 6x + 8$$

14) Factor: $x^2 - 4x - 21$

15) Factor: $3x^2 - 9x + 6$

6.3 MORE ON FACTORING TRINOMIALS

To factor $ax^2 + bx + c$,

By Grouping
Find m and n.

$mn = ac$

$ax^2 + bx + c$

$m + n = b$

By Trial and Error
Use FOIL in reverse.

Factor $3x^2 + 14x - 5$.

-15

$mn = -15,\ m + n = 14$

$m = -1$ and $n = 15$ ← Choice of m and n

By trial and error or by grouping,

$$3x^2 + 14x - 5 = (3x - 1)(x + 5).$$

16) Factor: $35x^2 + 2x - 1$

17) Factor: $16x^2 - 20x - 6$

6.4 SPECIAL FACTORING TECHNIQUES

Difference of Squares

$$x^2 - y^2 = (x + y)(x - y)$$

Perfect Square Trinomials

$$x^2 + 2xy + y^2 = (x + y)^2$$

$$x^2 - 2xy + y^2 = (x - y)^2$$

Difference of Cubes

$$x^3 - y^3 = (x - y)(x^2 + xy + y^2)$$

Sum of Cubes

$$x^3 + y^3 = (x + y)(x^2 - xy + y^2)$$

Factor.

$$4x^2 - 9 = (2x + 3)(2x - 3)$$

$$9x^2 + 6x + 1 = (3x + 1)^2$$

$$4x^2 - 20x + 25 = (2x - 5)^2$$

$$m^3 - 8 = m^3 - 2^3 = (m - 2)(m^2 + 2m + 4)$$

$$z^3 + 27 = z^3 + 3^3 = (z + 3)(z^2 - 3z + 9)$$

18) Factor: $9x^2 - 1$

19) Factor: $24x^3 - 81$

To find the value(s) for which a rational expression is undefined, set the denominator equal to 0 and solve the equation.

Find the values for which the expression $\frac{x-4}{x^2-16}$ is undefined.

$$x^2 - 16 = 0$$

$$(x-4)(x+4) = 0 \quad \text{Factor.}$$

$$x - 4 = 0 \quad \text{or} \quad x + 4 = 0 \quad \text{Zero-factor property}$$

$$x = 4 \quad \text{or} \quad x = -4$$

The rational expression is undefined for 4 and −4, so $x \neq 4$ and $x \neq -4$.

Writing a Rational Expression in Lowest Terms

Step 1 Factor the numerator and denominator.

Step 2 Use the fundamental property to divide out common factors.

Write $\frac{x^2-1}{(x-1)^2}$ in lowest terms.

$$\frac{x^2-1}{(x-1)^2} = \frac{(x-1)(x+1)}{(x-1)(x-1)}$$

$$= \frac{x+1}{x-1}$$

There are often several different equivalent forms of a rational expression.

Give four equivalent forms of $-\frac{x-1}{x+2}$.

Distribute the − sign in the numerator to get $\frac{-(x-1)}{x+2}$ or $\frac{-x+1}{x+2}$; do so in the denominator to get $\frac{x-1}{-(x+2)}$ or $\frac{x-1}{-x-2}$.

(continued)

20) Find the values for which the rational expression is undefined:

$$\frac{x^2 - 4x - 21}{x^2 + 5x}$$

21) Simplify:

$$\frac{x^2 + 5x - 14}{4 - x^2}$$

Multiplying or Dividing Rational Expressions

Step 1 Note the operation. If the operation is division, use the definition of division to rewrite as multiplication.

Step 2 Multiply numerators and multiply denominators.

Step 3 Factor numerators and denominators completely.

Step 4 Write in lowest terms, using the fundamental property.

Note: Steps 2 and 3 may be interchanged based on personal preference.

Multiply. $\frac{3x+9}{x-5} \cdot \frac{x^2-3x-10}{x^2-9}$

$$= \frac{(3x+9)(x^2-3x-10)}{(x-5)(x^2-9)}$$

$$= \frac{3(x+3)(x-5)(x+2)}{(x-5)(x+3)(x-3)}$$

$$= \frac{3(x+2)}{x-3}$$

Divide. $\frac{2x+1}{x+5} \div \frac{6x^2-x-2}{x^2-25}$

$$= \frac{2x+1}{x+5} \cdot \frac{x^2-25}{6x^2-x-2}$$ Multiply by the reciprocal.

$$= \frac{(2x+1)(x^2-25)}{(x+5)(6x^2-x-2)}$$

$$= \frac{(2x+1)(x+5)(x-5)}{(x+5)(2x+1)(3x-2)}$$

$$= \frac{x-5}{3x-2}$$

22) Multiply:

$$\frac{x^2+3x-10}{x^2-9} \quad \frac{x^3-27}{x^2+5x}$$

23) Divide:

$$\frac{9x^2-1}{9x^2+9x} \div \frac{9x^2+6x+1}{3x^2}$$

7.3 LEAST COMMON DENOMINATORS

Finding the LCD

Step 1 Factor each denominator into prime factors.

Step 2 List each different factor the greatest number of times it appears.

Step 3 Multiply the factors from Step 2 to get the LCD.

Find the LCD for $\frac{3}{k^2 - 8k + 16}$ and $\frac{1}{4k^2 - 16k}$.

$$\left.\begin{aligned} k^2 - 8k + 16 &= (k - 4)^2 \\ 4k^2 - 16k &= 4k(k - 4) \end{aligned}\right\} \text{Factor each denominator.}$$

$$\begin{aligned} \text{LCD} &= (k - 4)^2 \cdot 4 \cdot k \\ &= 4k(k - 4)^2 \end{aligned}$$

Writing a Rational Expression with a Specified Denominator

Step 1 Factor both denominators.

Find the numerator: $\frac{5}{2z^2 - 6z} = \frac{?}{4z^3 - 12z^2}$.

$$\frac{5}{2z(z - 3)} = \frac{?}{4z^2(z - 3)}$$

Step 2 Decide what factor(s) the denominator must be multiplied by in order to equal the specified denominator.

$2z(z - 3)$ must be multiplied by $2z$ in order to obtain $4z^2(z - 3)$.

Step 3 Multiply the rational expression by that factor divided by itself. (That is, multiply by 1.)

$$\frac{5}{2z(z - 3)} \cdot \frac{2z}{2z} = \frac{10z}{4z^2(z - 3)} = \frac{10z}{4z^3 - 12z^2}$$

(continued)

7.4 ADDING AND SUBTRACTING RATIONAL EXPRESSIONS

Adding Rational Expressions

Add. $\frac{2}{3m + 6} + \frac{m}{m^2 - 4}$

Step 1 Find the LCD.

$$\left.\begin{aligned} 3m + 6 &= 3(m + 2) \\ m^2 - 4 &= (m + 2)(m - 2) \end{aligned}\right\} \text{The LCD is } 3(m + 2)(m - 2).$$

Step 2 Rewrite each rational expression with the LCD as denominator.

$$= \frac{2(m - 2)}{3(m + 2)(m - 2)} + \frac{3m}{3(m + 2)(m - 2)}$$

Step 3 Add the numerators to get the numerator of the sum. The LCD is the denominator of the sum.

$$= \frac{2m - 4 + 3m}{3(m + 2)(m - 2)}$$

Step 4 Write in lowest terms.

$$= \frac{5m - 4}{3(m + 2)(m - 2)}$$

Subtracting Rational Expressions

Follow the same steps as for addition, but subtract in Step 3.

Subtract. $\frac{6}{k + 4} - \frac{2}{k}$ The LCD is $k(k + 4)$.

$$\begin{aligned} &= \frac{6k}{(k + 4)k} - \frac{2(k + 4)}{k(k + 4)} \\ &= \frac{6k - 2(k + 4)}{k(k + 4)} \\ &= \frac{6k - 2k - 8}{k(k + 4)} \\ &= \frac{4k - 8}{k(k + 4)}, \quad \text{or} \quad \frac{4(k - 2)}{k(k + 4)} \end{aligned}$$

Be careful with signs when subtracting the numerators.

24) Add:

$$\frac{1}{x+2} + \frac{2}{x^2 + 2x}$$

25) Subtract:

$$\frac{x-4}{x^2 - 3x + 2} - \frac{x+2}{x^2 - x}$$

7.5 COMPLEX FRACTIONS

Simplifying Complex Fractions

Method 1 Simplify the numerator and denominator separately. Then divide the simplified numerator by the simplified denominator.

Method 2 Multiply the numerator and denominator of the complex fraction by the LCD of all the denominators in the complex fraction. Write in lowest terms.

Simplify.

Method 1

$$\frac{\frac{1}{a} - a}{1 - a} = \frac{\frac{1}{a} - \frac{a^2}{a}}{1 - a} = \frac{\frac{1 - a^2}{a}}{1 - a}$$

$$= \frac{1 - a^2}{a} \div (1 - a)$$

$$= \frac{1 - a^2}{a} \cdot \frac{1}{1 - a}$$ Multiply by the reciprocal.

$$= \frac{(1 - a)(1 + a)}{a(1 - a)} = \frac{1 + a}{a}$$

Method 2

$$\frac{\frac{1}{a} - a}{1 - a} = \frac{\left(\frac{1}{a} - a\right)a}{(1 - a)a} = \frac{\frac{a}{a} - a^2}{(1 - a)a}$$

$$= \frac{1 - a^2}{(1 - a)a} = \frac{(1 + a)(1 - a)}{(1 - a)a}$$

$$= \frac{1 + a}{a}$$

8.1 EVALUATING ROOTS

If a is a positive real number, then

$\sqrt{a}$ is the positive square root of a;

$-\sqrt{a}$ is the negative square root of a; $\sqrt{0} = 0$.

If a is a negative real number, then $\sqrt{a}$ is not a real number.

$\sqrt{49} = 7$

$-\sqrt{81} = -9$

$\sqrt{-25}$ is not a real number.

If a is a positive rational number, then $\sqrt{a}$ is rational if a is a perfect square and $\sqrt{a}$ is irrational if a is not a perfect square.

$\sqrt{\frac{4}{9}}$, $\sqrt{16}$ are rational. $\sqrt{\frac{2}{3}}$, $\sqrt{21}$ are irrational.

Each real number has exactly one real cube root.

$\sqrt[3]{27} = 3$ $\quad \sqrt[3]{-8} = -2$

8.2 MULTIPLYING, DIVIDING, AND SIMPLIFYING RADICALS

Product Rule for Radicals

For nonnegative real numbers a and b,

$$\sqrt{a} \cdot \sqrt{b} = \sqrt{ab} \quad \text{and} \quad \sqrt{a \cdot b} = \sqrt{a} \cdot \sqrt{b}.$$

$\sqrt{5} \cdot \sqrt{7} = \sqrt{35}$

$\sqrt{8} \cdot \sqrt{2} = \sqrt{16} = 4$

$\sqrt{48} = \sqrt{16 \cdot 3} = \sqrt{16} \cdot \sqrt{3} = 4\sqrt{3}$

Quotient Rule for Radicals

If a and b are nonnegative real numbers and b is not 0, then

$$\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}} \quad \text{and} \quad \sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}.$$

$\frac{\sqrt{8}}{\sqrt{2}} = \sqrt{\frac{8}{2}} = \sqrt{4} = 2 \qquad \sqrt{\frac{25}{64}} = \frac{\sqrt{25}}{\sqrt{64}} = \frac{5}{8}$

If all indicated roots are real, then

$$\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{ab} \quad \text{and} \quad \frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}} \quad (b \neq 0).$$

$\sqrt[3]{5} \cdot \sqrt[3]{3} = \sqrt[3]{15} \qquad \frac{\sqrt[4]{12}}{\sqrt[4]{4}} = \sqrt[4]{\frac{12}{4}} = \sqrt[4]{3}$

26) Simplify:

$$\sqrt{14} \cdot \sqrt{21}$$

27) Simplify:

$$\sqrt[3]{-16x^2y^3z^4}$$

8.3 ADDING AND SUBTRACTING RADICALS

Add and subtract like radicals by using the distributive property. ***Only like radicals can be combined in this way.***

$$2\sqrt{5} + 4\sqrt{5} = (2+4)\sqrt{5} = 6\sqrt{5}$$

$$\sqrt{8} + \sqrt{32} = 2\sqrt{2} + 4\sqrt{2} = 6\sqrt{2}$$

28) Simplify: $\sqrt{7} + \sqrt{8}$

29) Simplify: $\sqrt{8} - \sqrt{27} + \sqrt{50} + \sqrt{48}$

8.4 RATIONALIZING THE DENOMINATOR

The denominator of a radical can be rationalized by multiplying both the numerator and denominator by a number that will eliminate the radical from the denominator.

$$\frac{2}{\sqrt{3}} = \frac{2 \cdot \sqrt{3}}{\sqrt{3} \cdot \sqrt{3}} = \frac{2\sqrt{3}}{3}$$

$$\sqrt[3]{\frac{5}{6}} = \frac{\sqrt[3]{5} \cdot \sqrt[3]{6^2}}{\sqrt[3]{6} \cdot \sqrt[3]{6^2}} = \frac{\sqrt[3]{180}}{6}$$

8.5 MORE SIMPLIFYING AND OPERATIONS WITH RADICALS

When appropriate, use the rules for adding and multiplying polynomials to simplify radical expressions.

$$\sqrt{6}(\sqrt{5} - \sqrt{7}) = \sqrt{30} - \sqrt{42}$$

$$(\sqrt{5} - \sqrt{3})(\sqrt{5} + \sqrt{3}) = 5 - 3 = 2$$

Any denominators with radicals should be rationalized.

$$\frac{3}{\sqrt{6}} = \frac{3\sqrt{6}}{6} = \frac{\sqrt{6}}{2}$$

If a radical expression contains two terms in the denominator and at least one of those terms is a square root radical, multiply both numerator and denominator by the conjugate of the denominator.

$$\frac{6}{\sqrt{7} - \sqrt{2}} = \frac{6}{\sqrt{7} - \sqrt{2}} \cdot \frac{\sqrt{7} + \sqrt{2}}{\sqrt{7} + \sqrt{2}}$$

$$= \frac{6(\sqrt{7} + \sqrt{2})}{7 - 2} \quad \text{Multiply fractions.}$$

$$= \frac{6(\sqrt{7} + \sqrt{2})}{5} \quad \text{Subtract.}$$

30) Rationalize:

$$\frac{2\sqrt{3} - 5}{\sqrt{24}}$$

31) Rationalize:

$$\frac{2\sqrt{3} + 5}{2\sqrt{3} - 5}$$

8.7 USING RATIONAL NUMBERS AS EXPONENTS

Assume that $a \geq 0$, m and n are integers, and $n > 0$. Then

$$a^{1/n} = \sqrt[n]{a}$$
$$a^{m/n} = \sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m$$
$$a^{-m/n} = \frac{1}{a^{m/n}} \quad (a \neq 0)$$

$$8^{1/3} = \sqrt[3]{8} = 2$$
$$(81)^{3/4} = \sqrt[4]{81^3} = \left(\sqrt[4]{81}\right)^3 = 3^3 = 27$$
$$36^{-3/2} = \frac{1}{36^{3/2}} = \frac{1}{(36^{1/2})^3} = \frac{1}{6^3} = \frac{1}{216}$$

9.4 COMPLEX NUMBERS

The imaginary unit is i, where

$$i = \sqrt{-1} \quad \text{and} \quad i^2 = -1.$$

For the positive number b, $\sqrt{-b} = i\sqrt{b}$.

$$\sqrt{-19} = i\sqrt{19}$$

Addition
Add complex numbers by adding the real parts and adding the imaginary parts.

Add: $(3 + 6i) + (-9 + 2i) = (3 - 9) + (6 + 2)i$
$= -6 + 8i$

Subtraction
To subtract complex numbers, change the number following the subtraction sign to its negative and add.

Subtract: $(5 + 4i) - (2 - 4i) = (5 + 4i) + (-2 + 4i)$
$= (5 - 2) + (4 + 4)i$
$= 3 + 8i$

Multiplication
Multiply complex numbers in the same way polynomials are multiplied. Replace i^2 with -1.

Multiply: $(7 + i)(3 - 4i)$
$= 7(3) + 7(-4i) + i(3) + i(-4i)$ FOIL method
$= 21 - 28i + 3i - 4i^2$
$= 21 - 25i - 4(-1)$ $i^2 = -1$
$= 21 - 25i + 4$
$= 25 - 25i$

Division
Divide complex numbers by multiplying the numerator and the denominator by the conjugate of the denominator.

Divide: $\frac{2}{6 + i} = \frac{2}{6 + i} \cdot \frac{6 - i}{6 - i}$
$= \frac{2(6 - i)}{36 - i^2}$
$= \frac{12 - 2i}{36 - (-1)}$ $i^2 = -1$
$= \frac{12 - 2i}{37}$
$= \frac{12}{37} - \frac{2}{37}i$ Standard form

Complex Solutions

A quadratic equation may have nonreal complex solutions. This occurs when the discriminant is negative. The quadratic formula will give complex solutions in such cases.

Solve for all complex solutions of $x^2 + x + 1 = 0$.

$$x = \frac{-1 \pm \sqrt{1^2 - 4(1)(1)}}{2(1)} \qquad a = 1,\ b = 1,\ c = 1$$

$$x = \frac{-1 \pm \sqrt{1 - 4}}{2}$$

$$x = \frac{-1 \pm \sqrt{-3}}{2}$$

$$x = \frac{-1 \pm i\sqrt{3}}{2} \qquad \sqrt{-b} = i\sqrt{b}$$

Solution set: $\left\{-\frac{1}{2} \pm \frac{\sqrt{3}}{2}i\right\}$

9.5 MORE ON GRAPHING QUADRATIC EQUATIONS; QUADRATIC FUNCTIONS

To graph $y = ax^2 + bx + c$,

Step 1 Find the vertex: $x = -\frac{b}{2a}$; find y by substituting this value for x in the equation.

Graph $y = 2x^2 - 5x - 3$.

$$x = -\frac{b}{2a} = -\frac{-5}{2(2)} = \frac{5}{4}$$

$$y = 2\left(\frac{5}{4}\right)^2 - 5\left(\frac{5}{4}\right) - 3$$

$$= 2\left(\frac{25}{16}\right) - \frac{25}{4} - 3$$

$$= \frac{25}{8} - \frac{50}{8} - \frac{24}{8} = -\frac{49}{8}$$

The vertex is $\left(\frac{5}{4}, -\frac{49}{8}\right)$.

Step 2 Find the y-intercept.

$$y = 2(0)^2 - 5(0) - 3 = -3$$

The y-intercept is $(0, -3)$.

Step 3 Find the x-intercepts (if they exist).

$$0 = 2x^2 - 5x - 3$$

$$0 = (2x + 1)(x - 3)$$

$$2x + 1 = 0 \quad \text{or} \quad x - 3 = 0$$

$$2x = -1 \quad \text{or} \quad x = 3$$

$$x = -\frac{1}{2} \quad \text{or} \quad x = 3$$

The x-intercepts are $\left(-\frac{1}{2}, 0\right)$ and $(3, 0)$.

Step 4 Plot the intercepts and the vertex.

Step 5 Find and plot additional ordered pairs near the vertex and intercepts as needed.

x	y
$-\frac{1}{2}$	0
0	-3
$\frac{5}{4}$	$-\frac{49}{8}$
2	-5
3	0