Exercise Set 4.9

1. Use matrix multiplication to find the reflection of $(-1, 2)$ about the

(a) x-axis. (b) y-axis. (c) line $y = x$.

2. Use matrix multiplication to find the reflection of (a, b) about the

(a) x-axis. (b) y-axis. (c) line $y = x$.

3. Use matrix multiplication to find the reflection of $(2, -5, 3)$ about the

(a) xy-plane. (b) xz-plane. (c) yz-plane.

4. Use matrix multiplication to find the reflection of (a, b, c) about the

(a) xy-plane. (b) xz-plane. (c) yz-plane.

5. Use matrix multiplication to find the orthogonal projection of $(2, -5)$ onto the

(a) x-axis. (b) y-axis.

6. Use matrix multiplication to find the orthogonal projection of (a, b) onto the

(a) x-axis. (b) y-axis.

7. Use matrix multiplication to find the orthogonal projection of $(-2, 1, 3)$ onto the

(a) xy-plane. (b) xz-plane. (c) yz-plane.

8. Use matrix multiplication to find the orthogonal projection of (a, b, c) onto the

(a) xy-plane. (b) xz-plane. (c) yz-plane.

9. Use matrix multiplication to find the image of the vector $(3, -4)$ when it is rotated about the origin through an angle of

(a) $\theta = 30^\circ$. (b) $\theta = -60^\circ$.

(c) $\theta = 45^\circ$. (d) $\theta = 90^\circ$.

10. Use matrix multiplication to find the image of the nonzero vector $\mathbf{v} = (v_1, v_2)$ when it is rotated about the origin through

(a) a positive angle α. (b) a negative angle $-\alpha$.

11. Use matrix multiplication to find the image of the vector $(2, -1, 2)$ if it is rotated

(a) 30° clockwise about the positive x-axis.

(b) 30° counterclockwise about the positive y-axis.

(c) 45° clockwise about the positive y-axis.

(d) 90° counterclockwise about the positive z-axis.

12. Use matrix multiplication to find the image of the vector $(2, -1, 2)$ if it is rotated

(a) 30° counterclockwise about the positive x-axis.

(b) 30° clockwise about the positive y-axis.

(c) 45° counterclockwise about the positive y-axis.

(d) 90° clockwise about the positive z-axis.

13. (a) Use matrix multiplication to find the contraction of $(-1, 2)$ with factor $k = \frac{1}{2}$.

(b) Use matrix multiplication to find the dilation of $(-1, 2)$ with factor $k = 3$.

14. (a) Use matrix multiplication to find the contraction of (a, b) with factor $k = 1/\alpha$, where $\alpha > 1$.

(b) Use matrix multiplication to find the dilation of (a, b) with factor $k = \alpha$, where $\alpha > 1$.

15. (a) Use matrix multiplication to find the contraction of $(2, -1, 3)$ with factor $k = \frac{1}{4}$.

(b) Use matrix multiplication to find the dilation of $(2, -1, 3)$ with factor $k = 2$.

16. (a) Use matrix multiplication to find the contraction of (a, b, c) with factor $k = 1/\alpha$, where $\alpha > 1$.

(b) Use matrix multiplication to find the dilation of (a, b, c) with factor $k = \alpha$, where $\alpha > 1$.

17. (a) Use matrix multiplication to find the compression of $(-1, 2)$ in the x-direction with factor $k = \frac{1}{2}$.

(b) Use matrix multiplication to find the compression of $(-1, 2)$ in the y-direction with factor $k = \frac{1}{2}$.

18. (a) Use matrix multiplication to find the expansion of $(-1, 2)$ in the x-direction with factor $k = 3$.

(b) Use matrix multiplication to find the expansion of $(-1, 2)$ in the y-direction with factor $k = 3$.

19. (a) Use matrix multiplication to find the compression of (a, b) in the x-direction with factor $k = 1/\alpha$, where $\alpha > 1$.

(b) Use matrix multiplication to find the expansion of (a, b) in the y-direction with factor $k = \alpha$, where $\alpha > 1$.

20. Based on Table 9, make a conjecture about the standard matrices for the compressions with factor k in the directions of the coordinate axes in R^3.

▶ **Exercises 21–22** Using Example 2 as a model, describe the matrix operator whose standard matrix is given, and then show in a coordinate system its effect on the unit square. ◀

21. (a) $A_1 = \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{bmatrix}$ (b) $A_2 = \begin{bmatrix} 1 & 0 \\ 0 & \frac{1}{2} \end{bmatrix}$

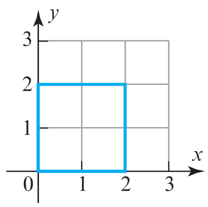
(c) $A_3 = \begin{bmatrix} 1 & 0 \\ \frac{1}{2} & 1 \end{bmatrix}$ (d) $A_4 = \begin{bmatrix} 1 & 0 \\ -\frac{1}{2} & 1 \end{bmatrix}$

22. (a) $A_1 = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$ (b) $A_2 = \begin{bmatrix} 1 & 0 \\ 0 & 3 \end{bmatrix}$

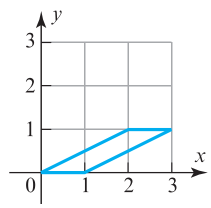
(c) $A_3 = \begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix}$ (d) $A_4 = \begin{bmatrix} 1 & 0 \\ -3 & 1 \end{bmatrix}$

▶ In each part of Exercises 23–24, the effect of some matrix operator on the unit square is shown. Find the standard matrix for an operator with that effect. ◀

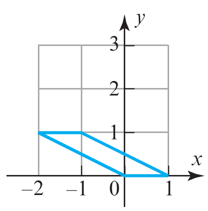
23. (a)



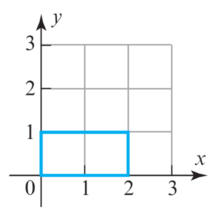
(b)



24. (a)



(b)



▶ In Exercises 25–26, find the standard matrix for the orthogonal projection of R^2 onto the stated line, and then use that matrix to find the orthogonal projection of the given point onto that line. ◀

25. The orthogonal projection of (3, 4) onto the line that makes an angle of $\pi/3$ (= 60°) with the positive x-axis.

26. The orthogonal projection of (1, 2) onto the line that makes an angle of $\pi/4$ (= 45°) with the positive x-axis.

▶ In Exercises 27–28, find the standard matrix for the reflection of R^2 about the stated line, and then use that matrix to find the reflection of the given point about that line. ◀

27. The reflection of (3, 4) about the line that makes an angle of $\pi/3$ (= 60°) with the positive x-axis.

28. The reflection of (1, 2) about the line that makes an angle of $\pi/4$ (= 45°) with the positive x-axis.

29. For each reflection operator in Table 2 use the standard matrix to compute $T(1, 2, 3)$, and convince yourself that your result makes sense geometrically.

30. For each orthogonal projection operator in Table 4 use the standard matrix to compute $T(1, 2, 3)$, and convince yourself that your result makes sense geometrically.

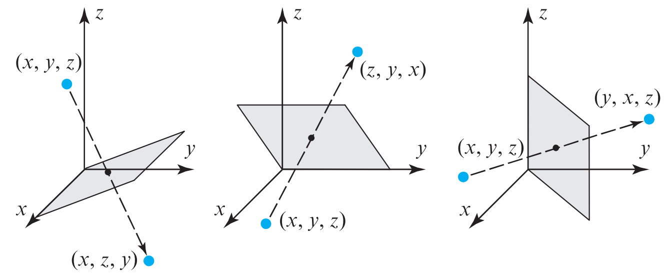
31. Find the standard matrix for the operator $T: R^3 \to R^3$ that

(a) rotates each vector 30° counterclockwise about the z-axis (looking along the positive z-axis toward the origin).

(b) rotates each vector 45° counterclockwise about the x-axis (looking along the positive x-axis toward the origin).

(c) rotates each vector 90° counterclockwise about the y-axis (looking along the positive y-axis toward the origin).

32. In each part of the accompanying figure, find the standard matrix for the pictured operator.



(a) (b) (c)

▲ Figure Ex-32

33. Use Formula (3) to find the standard matrix for a rotation of 180° about the axis determined by the vector $\mathbf{v} = (2, 2, 1)$. [*Note:* Formula (3) requires that the vector defining the axis of rotation have length 1.]

34. Use Formula (3) to find the standard matrix for a rotation of $\pi/2$ radians about the axis determined by $\mathbf{v} = (1, 1, 1)$. [*Note:* Formula (3) requires that the vector defining the axis of rotation have length 1.]

35. Use Formula (3) to derive the standard matrices for the rotations about the x-axis, the y-axis, and the z-axis through an angle of 90° in R^3.

36. Show that the standard matrices listed in Tables 1 and 3 are special cases of Formulas (4) and (6).

37. In a sentence, describe the geometric effect of multiplying a vector $\mathbf{x}$ by the matrix

$$A = \begin{bmatrix} \cos^2\theta - \sin^2\theta & -2\sin\theta\cos\theta \\ 2\sin\theta\cos\theta & \cos^2\theta - \sin^2\theta \end{bmatrix}$$

38. If multiplication by A rotates a vector $\mathbf{x}$ in the xy-plane through an angle θ, what is the effect of multiplying $\mathbf{x}$ by A^T? Explain your reasoning.

39. Let $\mathbf{x}_0$ be a nonzero column vector in R^2, and suppose that $T: R^2 \to R^2$ is the transformation defined by the formula $T(\mathbf{x}) = \mathbf{x}_0 + R_\theta \mathbf{x}$, where R_θ is the standard matrix of the rotation of R^2 about the origin through the angle θ. Give a geometric description of this transformation. Is it a matrix transformation? Explain.

40. In R^3 the ***orthogonal projections*** onto the x-axis, y-axis, and z-axis are

$$T_1(x, y, z) = (x, 0, 0), \quad T_2(x, y, z) = (0, y, 0),$$
$$T_3(x, y, z) = (0, 0, z)$$

respectively.

(a) Show that the orthogonal projections onto the coordinate axes are matrix operators, and then find their standard matrices.