Exercise Set 6.6 (page 399)

1. (a) $1+\pi-2\sin x-\sin 2x$ (b) $1+\pi-\frac{2}{1}\sin x-\frac{2}{2}\sin(2x)-\cdots-\frac{2}{n}\sin(nx)$

3. (a) $\frac{e^x}{e-1}-\frac{1}{2}$ (b) $\frac{7e-19}{12e-12}\approx 0.00136$

5. (a) $\frac{3x}{\pi}$ (b) $1-\frac{6}{\pi^2}\approx 0.392$ 9. $\frac{1}{2}+\sum_{k=1}^{\infty}\frac{1}{k\pi}\left(1-(-1)^k\right)\sin kx$

True/False 6.6

(a) False **(b)** True **(c)** True **(d)** False **(e)** True

Chapter 6 Supplementary Exercises (page 399)

1. (a) $(0, a, a, 0)$ with $a\neq 0$ (b) $\pm\left(0, \frac{2}{\sqrt{5}}, \frac{1}{\sqrt{5}}, 0\right)$

3. (a) The subspace of all matrices in M_{22} with zeros on the main diagonal.
   (b) The subspace of all 2×2 skew-symmetric matrices.

7. $\pm\left(\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}\right)$ 9. No 11. (b) θ approaches $\frac{\pi}{2}$ 17. No

Exercise Set 7.1 (page 407)

1. (a) Orthogonal; $A^{-1}=\begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix}$ (b) Orthogonal; $A^{-1}=\begin{bmatrix}\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}}\\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}}\end{bmatrix}$

3. (a) Not orthogonal (b) Orthogonal; $A^{-1}=\begin{bmatrix}-\frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}}\\ \frac{1}{\sqrt{6}} & -\frac{2}{\sqrt{6}} & \frac{1}{\sqrt{6}}\\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}}\end{bmatrix}$ 7. $T_A(\mathbf{x})=\begin{bmatrix}-\frac{23}{5}\\ \frac{18}{25}\\ \frac{101}{25}\end{bmatrix}$; $\|T_A(\mathbf{x})\|=\|\mathbf{x}\|=\sqrt{38}$

9. Yes 11. $a^2+b^2=\frac{1}{2}$ 13. (a) $\begin{bmatrix}-1+3\sqrt{3}\\ 3+\sqrt{3}\end{bmatrix}$ (b) $\begin{bmatrix}\frac{5}{2}-\sqrt{3}\\ 1+\frac{5}{2}\sqrt{3}\end{bmatrix}$ 15. (a) $\begin{bmatrix}\frac{1}{\sqrt{2}}\\ \frac{3}{\sqrt{2}}\\ 5\end{bmatrix}$ (b) $\begin{bmatrix}-\frac{5}{\sqrt{2}}\\ \frac{7}{\sqrt{2}}\\ -3\end{bmatrix}$

17. (a) $\begin{bmatrix}-\frac{1}{2}-\frac{5\sqrt{3}}{2}\\ 2\\ -\frac{\sqrt{3}}{2}+\frac{5}{2}\end{bmatrix}$ (b) $\begin{bmatrix}\frac{1}{2}-\frac{3\sqrt{3}}{2}\\ 6\\ -\frac{\sqrt{3}}{2}-\frac{3}{2}\end{bmatrix}$ 19. $\begin{bmatrix}1 & 0 & 0\\ 0 & \cos\theta & \sin\theta\\ 0 & -\sin\theta & \cos\theta\end{bmatrix}$

21. (a) Rotations about the origin, reflections about any line through the origin, and any combination of these
    (b) Rotations about the origin, dilations, contractions, reflections about lines through the origin, and combinations of these
    (c) No; dilations and contractions

23. (a) $(\mathbf{p})_S=\left(\frac{5}{\sqrt{3}}, \sqrt{2}, \frac{\sqrt{2}}{\sqrt{3}}\right)$, $(\mathbf{q})_S=\left(-\frac{2}{\sqrt{3}}, 2\sqrt{2}, -\frac{\sqrt{2}}{\sqrt{3}}\right)$
    (b) $\|\mathbf{p}\|=\sqrt{11}$, $d(\mathbf{p},\mathbf{q})=\sqrt{21}$, $\langle\mathbf{p},\mathbf{q}\rangle=0$

True/False 7.1

(a) False **(b)** False **(c)** False **(d)** False **(e)** True **(f)** True **(g)** True **(h)** True

Exercise Set 7.2 (page 416)

1. $\lambda^2 - 5\lambda = 0$; $\lambda = 0$: one-dimensional; $\lambda = 5$: one-dimensional

3. $\lambda^3 - 3\lambda^2 = 0$; $\lambda = 3$: one-dimensional; $\lambda = 0$: two-dimensional

5. $\lambda^4 - 8\lambda^3 = 0$; $\lambda = 0$: three-dimensional; $\lambda = 8$: one-dimensional

7. $P = \begin{bmatrix} -\frac{2}{\sqrt{7}} & \frac{\sqrt{3}}{\sqrt{7}} \\ \frac{\sqrt{3}}{\sqrt{7}} & \frac{2}{\sqrt{7}} \end{bmatrix}$; $P^{-1}AP = \begin{bmatrix} 3 & 0 \\ 0 & 10 \end{bmatrix}$

9. $P = \begin{bmatrix} -\frac{4}{5} & 0 & \frac{3}{5} \\ 0 & 1 & 0 \\ \frac{3}{5} & 0 & \frac{4}{5} \end{bmatrix}$; $P^{-1}AP = \begin{bmatrix} 25 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & -50 \end{bmatrix}$

11. $P = \begin{bmatrix} -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} \\ 0 & \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} \end{bmatrix}$; $P^{-1}AP = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

13. $P = \begin{bmatrix} -\frac{4}{5} & 0 & \frac{3}{5} & 0 \\ \frac{3}{5} & 0 & \frac{4}{5} & 0 \\ 0 & -\frac{4}{5} & 0 & \frac{3}{5} \\ 0 & \frac{3}{5} & 0 & \frac{4}{5} \end{bmatrix}$; $P^{-1}AP = \begin{bmatrix} -25 & 0 & 0 & 0 \\ 0 & -25 & 0 & 0 \\ 0 & 0 & 25 & 0 \\ 0 & 0 & 0 & 25 \end{bmatrix}$

15. $(2)\begin{bmatrix} -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix}\begin{bmatrix} -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} + (4)\begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix}\begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} = (2)\begin{bmatrix} \frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} \end{bmatrix} + (4)\begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix}$

17. $(-4)\begin{bmatrix} -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \\ 0 \end{bmatrix}\begin{bmatrix} -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \end{bmatrix} + (-4)\begin{bmatrix} -\frac{1}{\sqrt{3}} \\ -\frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \end{bmatrix}\begin{bmatrix} -\frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \end{bmatrix} + (2)\begin{bmatrix} \frac{1}{\sqrt{6}} \\ \frac{1}{\sqrt{6}} \\ \frac{2}{\sqrt{6}} \end{bmatrix}\begin{bmatrix} \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} & \frac{2}{\sqrt{6}} \end{bmatrix}$

$= (-4)\begin{bmatrix} \frac{1}{2} & -\frac{1}{2} & 0 \\ -\frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 0 \end{bmatrix} + (-4)\begin{bmatrix} \frac{1}{3} & \frac{1}{3} & -\frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} & -\frac{1}{3} \\ -\frac{1}{3} & -\frac{1}{3} & \frac{1}{3} \end{bmatrix} + (2)\begin{bmatrix} \frac{1}{6} & \frac{1}{6} & \frac{1}{3} \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} & \frac{2}{3} \end{bmatrix}$

19. $\begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 4 \\ 0 & 4 & 3 \end{bmatrix}$ **21.** Yes **23. (a)** $\begin{bmatrix} \frac{\sqrt{2}-1}{4-2\sqrt{2}} \\ \frac{1}{4-2\sqrt{2}} \end{bmatrix}, \begin{bmatrix} \frac{-\sqrt{2}-1}{4+2\sqrt{2}} \\ \frac{1}{4+2\sqrt{2}} \end{bmatrix}$ **(b)** $\begin{bmatrix} \frac{-1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix}, \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix}$

True/False 7.2

(a) True **(b)** True **(c)** False **(d)** True **(e)** True **(f)** True **(g)** True

Exercise Set 7.3 (page 427)

1. (a) $[x_1 \quad x_2]\begin{bmatrix} 3 & 0 \\ 0 & 7 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ **(b)** $[x_1 \quad x_2]\begin{bmatrix} 4 & -3 \\ -3 & -9 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ **(c)** $[x_1 \quad x_2 \quad x_3]\begin{bmatrix} 9 & 3 & -4 \\ 3 & -1 & \frac{1}{2} \\ -4 & \frac{1}{2} & 4 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$

3. $2x^2 + 5y^2 - 6xy$ **5.** $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}\begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$; $Q = 3y_1^2 + y_2^2$

7. $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -\frac{2}{3} & \frac{2}{3} & -\frac{1}{3} \\ \frac{2}{3} & \frac{1}{3} & -\frac{2}{3} \\ \frac{1}{3} & \frac{2}{3} & \frac{2}{3} \end{bmatrix}\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$; $Q = y_1^2 + 4y_2^2 + 7y_3^2$