15. $P = \begin{bmatrix} \frac{-1-i}{\sqrt{6}} & \frac{1+i}{\sqrt{3}} \\ \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} \end{bmatrix}$; $P^{-1}AP = \begin{bmatrix} 2 & 0 \\ 0 & 8 \end{bmatrix}$ **17.** $P = \begin{bmatrix} 0 & 0 & 1 \\ \frac{1-i}{\sqrt{3}} & \frac{-1+i}{\sqrt{6}} & 0 \\ \frac{1}{\sqrt{3}} & \frac{2}{\sqrt{6}} & 0 \end{bmatrix}$; $P^{-1}AP = \begin{bmatrix} -2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 5 \end{bmatrix}$

19. $\begin{bmatrix} 0 & i & 2-3i \\ i & 0 & 1 \\ -2-3i & -1 & 4i \end{bmatrix}$ **27.** **(c)** B and C must commute **35.** $\begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{i}{\sqrt{2}} \\ \frac{i}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix}$

True/False 7.5

(a) False **(b)** False **(c)** True **(d)** False **(e)** False

Chapter 7 Supplementary Exercises (page 445)

1. **(a)** $\begin{bmatrix} \frac{3}{5} & -\frac{4}{5} \\ \frac{4}{5} & \frac{3}{5} \end{bmatrix}^{-1} = \begin{bmatrix} \frac{3}{5} & \frac{4}{5} \\ -\frac{4}{5} & \frac{3}{5} \end{bmatrix}$ **(b)** $\begin{bmatrix} \frac{4}{5} & 0 & -\frac{3}{5} \\ -\frac{9}{25} & \frac{4}{5} & -\frac{12}{25} \\ \frac{12}{25} & \frac{3}{5} & \frac{16}{25} \end{bmatrix}^{-1} = \begin{bmatrix} \frac{4}{5} & -\frac{9}{25} & \frac{12}{25} \\ 0 & \frac{4}{5} & \frac{3}{5} \\ -\frac{3}{5} & -\frac{12}{25} & -\frac{16}{25} \end{bmatrix}$

5. $P = \begin{bmatrix} -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \end{bmatrix}$; $P^TAP = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

7. Positive definite **9.** **(a)** Parabola **(b)** Parabola

13. Two possible solutions: $a = 0, b = \sqrt{\frac{2}{3}}, c = -\frac{1}{\sqrt{3}}$ and $a = 0, b = -\sqrt{\frac{2}{3}}, c = \frac{1}{\sqrt{3}}$

Exercise Set 8.1 (page 456)

1. **(a)** Nonlinear

(b) Linear; kernel consists of all matrices of the form $\begin{bmatrix} a & b \\ c & -a \end{bmatrix}$

(c) Linear; kernel consists of all matrices of the form $\begin{bmatrix} 0 & b \\ -b & 0 \end{bmatrix}$

3. Nonlinear **5.** Linear; kernel consists of all 2×2 matrices whose rows are orthogonal to all columns of B

7. **(a)** Linear; $\ker(T) = \{0\}$ **(b)** Nonlinear **9.** Linear; $\ker(T) = \{(0, 0, 0, \ldots)\}$ **11.** (a) and (d)

13. **(a)** 2 **(b)** 4 **(c)** $mn - 3$ **(d)** 1 **15.** **(a)** $\begin{bmatrix} 3 & 6 \\ -12 & 9 \end{bmatrix}$ **(b)** $\text{rank}(T) = 4$; $\text{nullity}(T) = 0$

17. **(a)** $(1, 0, 1)$ **(b)** $\ker(T) = \{0\}$ **(c)** $R(T) = R^3$ **19.** $T(x_1, x_2) = (-4x_1 + 5x_2, x_1 - 3x_2)$; $T(5, -3) = (-35, 14)$

19. $T(x_1, x_2, x_3) = (-x_1 + 4x_2 - x_3, 5x_1 - 5x_2 - x_3, x_1 + 3x_3)$; $T(2, 4, -1) = (15, -9, -1)$

23. **(b)** $\{x, x^2\}$ **(c)** $\{5, x^2\}$

25. **(a)** $\ker(D)$ consists of all constant polynomials

(b) $\ker(J)$ consists of all polynomials of the form a_1x

27. **(a)** $T(f(x)) = f^{(4)}(x)$

(b) $T(f(x)) = f^{(n+1)}(x)$

29. **(a)** The origin, a line through the origin, a plane through the origin, or the entire space R^3
(b) The origin, a line through the origin, a plane through the origin, or the entire space R^3

31. $(-10, -7, 6)$

True/False 8.1

(a) True **(b)** False **(c)** True **(d)** False **(e)** True **(f)** True **(g)** False **(h)** False **(i)** False

Exercise Set 8.2 (page 464)

1. **(a)** $\ker(T) = \{\mathbf{0}\}$; T is one-to-one **(b)** $\ker(T) = \{\mathbf{0}\}$; T is one-to-one **(c)** $\ker(T) = \{\text{span}(0, 1, 1)\}$; T is not one-to-one

3. **(a)** nullity$(A) = 1$; not one-to-one **(b)** nullity$(A) = 1$; not one-to-one

5. **(a)** One-to-one **(b)** One-to-one **(c)** Not one-to-one

7. For example, $T(1 - x^2) = (0, 0)$; T is onto

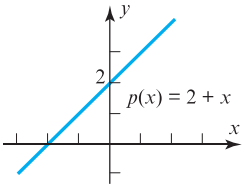
9. No; T is not one-to-one because $\ker(T) \neq \{\mathbf{0}\}$ as $T(\mathbf{a}) = \mathbf{a} \times \mathbf{a} = \mathbf{0}$

11. $(T_2 \circ T_1)(x, y) = (2x - 3y, 2x + 3y)$ **13.** $(T_3 \circ T_2 \circ T_1)(x, y) = (3x - 2y, x)$

15. **(a)** $a + d$ **(b)** $(T_2 \circ T_1)(A)$ does not exist because $T_1(A)$ is not a 2×2 matrix

17. $a_0x + a_1x(x + 1) + a_2x(x + 1)^2$

19. **(a)** $(1, -1)$ **(d)** $T^{-1}(2, 3) = 2 + x$



21. **(a)** all the a_i's must be nonzero **(b)** $T^{-1}(x_1, x_2, \ldots, x_n) = \left(\frac{1}{a_1}x_1, \frac{1}{a_2}x_2, \ldots, \frac{1}{a_n}x_n\right)$

23. **(a)** $T_1^{-1}(p(x)) = \frac{1}{x}p(x)$; $T_2^{-1}(p(x)) = p(x - 1)$; $(T_1^{-1} \circ T_2^{-1})(p(x)) = \frac{1}{x}p(x - 1)$

25. $T_2(\mathbf{v}) = \frac{1}{4}\mathbf{v}$ **31.** Since $\ker(J) \neq \{\mathbf{0}\}$, J is not one-to-one.

True/False 8.2

(a) True **(b)** False **(c)** True **(d)** True **(e)** False **(f)** True

Exercise Set 8.3 (page 471)

1. Isomorphism **3.** Isomorphism **5.** Not an isomorphism **7.** Isomorphism

9. **(a)** $T\left(\begin{bmatrix} a & b & c \\ b & d & e \\ c & e & f \end{bmatrix}\right) = \begin{bmatrix} a \\ b \\ c \\ d \\ e \\ f \end{bmatrix}$ **(b)** $T_1\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$; $T_2\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = \begin{bmatrix} a \\ c \\ b \\ d \end{bmatrix}$

11. Isomorphism **13.** $\dim(W) = 3$; $(-r - s - t, r, s, t) \rightarrow (r, s, t)$ is an isomorphism between W and R^3

15. Isomorphism **17.** Yes **19.** No

True/False 8.3

(a) False **(b)** True **(c)** False **(d)** True **(e)** True **(f)** True