1. (17 points) Write the following expressions so the exponents are positive. Leave answers in exponential form.

a. (3 points) $\frac{x^{-4}}{2x^{-8}}$

$$= x^{-4} \cdot \frac{1}{2} \cdot \frac{1}{x^{-8}}$$

$$= \frac{1}{x^4} \cdot \frac{1}{2} \cdot \frac{x^8}{1}$$

$$= \frac{x^8}{x^4} \cdot \frac{1}{2}$$

$$= \boxed{\frac{x^4}{2}}$$

c. (3 points) $(3t)^3(3t^{-8})$

$$= (3^3t^3)(3t^{-8})$$

$$= 3^3 \cdot 3 \cdot t^3 \cdot t^{-8}$$

$$= 3^4 \cdot t^{-5}$$

$$= \boxed{\frac{3^4}{t^5}}$$

b. (5 points) $\frac{(xy)^{-4}}{x^3y^{-5}}$

$$= \frac{x^{-4}y^{-4}}{x^3y^{-5}}$$

$$= \frac{1}{x^3x^4} \cdot \frac{y^5}{y^4}$$

$$= \boxed{\frac{y}{x^7}}$$

d. (6 points) $\frac{(6r^{-1})^2(2r^{-4})}{r^{-5}(r^2)^{-3}}$

$$= \frac{6^2 \cdot r^{-2} \cdot 2 \cdot r^{-4}}{r^{-5} \cdot r^{-6}}$$

$$= \frac{6^2 \cdot 2 \cdot \cancel{r^{-6}}}{r^{-5} \cdot \cancel{r^{-6}}}$$

$$= 6^2 \cdot 2 r^5$$

$$= \boxed{72r^5}$$

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2. (11 points) Simplify the following polynomials.
   a. (3 points) $(12y^4 - 7y^3 + 8y) + (4y^4 + 9y^3 - 6y^2 + 11)$

$$= 12y^4 + 4y^4 - 7y^3 + 9y^3 - 6y^2 + 8y + 11$$

$$= \boxed{16y^4 + 2y^3 - 6y^2 + 8y + 11}$$

   b. (3 points) $(12y^4 - 7y^3 + 8y) - (4y^4 + 9y^3 - 6y^2 + 11)$

$$= 12y^4 - 4y^4 - 7y^3 - 9y^3 + 6y^2 + 8y - 11$$

$$= \boxed{8y^4 - 16y^3 + 6y^2 + 8y - 11}$$

   c. (5 points) $(5x^3y^2 - 3xy^5 + 12x^2) - (-9x^2 - 8x^3y^2 + 2xy^5)$

$$= 5x^3y^2 + 8x^3y^2 - 3xy^5 - 2xy^5 + 12x^2 + 9x^2$$

$$= \boxed{13x^3y^2 - 5xy^5 + 21x^2}$$

3. (14 points) Find each product. Simplify your answer as much as possible.

a. (4 points) $(5t)(8t^7 - 5t^5 - 12t^2 - 3)$

$= (5t)(8t^7) + (5t)(-5t^5) + (5t)(-12t^2) + (5t)(-3)$

$= \boxed{40t^8 - 25t^6 - 60t^3 - 15t}$

b. (4 points) $(x - 5)(3y + 9)$

$= \boxed{3xy + 9x - 15y - 45}$

c. (6 points) $(a - 2)(5a^3 - 9a + 14)$

$= 5a^3(a-2) - (9a)(a-2) + 14(a-2)$

$= 5a^4 - 10a^3 - 9a^2 + 18a + 14a - 28$

$= \boxed{5a^4 - 10a^3 - 9a^2 + 32a - 28}$

4. (11 points) Perform each division.

a. (5 points) $\frac{\cancel{p}^2+2\cancel{p}+20}{\cancel{p}+6}$ $\frac{p^2+2p+20}{p+6}$ $= \boxed{p-4+\frac{44}{p+6}}$

$$
\begin{array}{r}
p-4 \\
p+6 \overline{) p^2+2p+20} \\
-(p^2+6p) \\
\hline
-4p+20 \\
-(-4p-24) \\
\hline
44
\end{array}
$$

b. (6 points) $\frac{27r^4-36r^3-6r^2+26r-24}{r-3}$ $= \boxed{27r^3+45r^2+129r+413+\frac{1215}{r-3}}$

$$
\begin{array}{r}
27r^3+45r^2+129r+413 \\
r-3 \overline{) 27r^4-36r^3-6r^2+26r-24} \\
-(27r^4-81r^3) \\
\hline
45r^3-6r^2 \\
-(45r^2-135r^2) \\
\hline
129r^2+26r \\
-(129r^2-387r) \\
\hline
\cancel{361}r-24 \\
413 \\
-(413r-1239) \\
\hline
1215
\end{array}
$$

5. (10 points) Perform the indicated operation.

a. (4 points) $\frac{5t^8-30t+15}{5t^3}$

$$= \frac{5t^8}{5t^3} - \frac{30t}{5t^3} + \frac{15}{5t^3}$$

$$= \boxed{t^5 - \frac{6}{t^2} + \frac{3}{t^3}}$$

b. (5 points) $\frac{-10m^4n+5m^3n^2+6m^2n^4}{5m^2n}$

$$= \frac{-10m^4n}{5m^2 \cdot n} + \frac{5m^3n^2}{5m^2n} + \frac{6m^2n^4}{5m^2n}$$

$$= \boxed{-2m^2 + mn + \frac{6n^3}{5}}$$

6. (11 points) Find the greatest common factor (GCF) of each expression. Rewrite the expression with the GCF.

a. (5 points) $60z^4 + 70z^8 + 90z^9 = (\ 10z^4\ )(\ 6 + 7z^4 + 9z^5\ )$

$60z^4 = 2^2 \cdot 3 \cdot 5\ z^4$

$70z^8 = 2 \cdot 5 \cdot 7 \cdot z^8$

$90z^9 = 2 \cdot 3^2 \cdot 5\ z^9$

GCF: $2 \cdot 5z^4$

b. (6 points) $100a^3z^5 + 60a^4z^4 - 85a^5z^2 = (5a^3z^2)(\ 20z^3 + 12az^2 - 17a^2)$

$100a^3z^5 = 2^2 5^2 a^3 z^5$

$60a^4z^4 = 2^2 \cdot 3 \cdot 5 \cdot z^4$

$-85a^5z^2 = -5 \cdot 17 a^5 z^2$

7. (16 points) Factor each expression completely.
   a. (5 points) $y^2 - 13y + 40$
   b. (6 points) $3x^4 + 30x^3 + 48x^2$
   c. (5 points) $n^2 - 64$

a) $y^2 - 13y + 40$

p: 40 s: -13

factors: 2 , 20
4 , 10
$-5, -8$ ✓

$\boxed{\cancel{(y+5)(y+8)}}$

$\boxed{(y-5)(y-8)}$

b) $3x^4 + 30x^3 + 48x^2$

$3x^2(\underbrace{x^2 + 10x + 16}_{\text{factor}})$ →

product : 16
sum : 10

factors
2 , 8

$(x+2)(x+8)$

$\boxed{3x^2(x+2)(x+8)}$

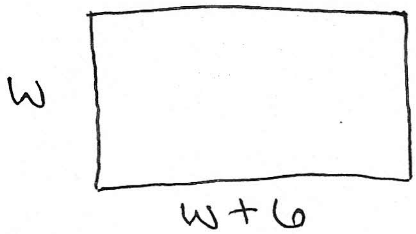
c) $n^2 - 64$ Difference of squares

$\boxed{(n-8)(n+8)}$

or product $= -64$
sum $= 0$

factors
$-8, 8$

8. (10 points) The length of a rug is 6 ft more than the width. The area is 40 ft^2. Find the length and width of the rug.



$w(w+6) = 40$

$w^2 + 6w = 40$

$w^2 + 6w - 40 = 0$

product -40
sum 6

factors

-4, 10

$(w-4)(w+10) = 0$

$w - 4 = 0$ $\quad$ $w + 10 = 0$

$\boxed{w = 4}$ $\quad$ $w = -10$

can't use negative length

$l = w + 6$

$\boxed{l = 10}$

Extra. (10 points) Factor and simplify the expression completely. $(m+n)^2 - (m-n)^2$

This is of the form $a^2 - b^2$

$= (a-b)(a+b)$ $\quad$ $a = m+n$, $b = m-n$

$\Rightarrow [(m+n) - (m-n)] \cdot [(m+n) + (m-n)]$

$= [2n][2m] = \boxed{4nm}$