1. (10 points) Write the rational expressions in lowest terms:

a. (5 points) $\frac{6a^2+a-2}{2a^2-3a+1}$

Factor Num & Denominator

$$\frac{6a^2+4a-3a-2}{2a^2-2a-a+1}$$

$$= \frac{2a(3a+2)+(-1)(3a+2)}{2a(a-1)+(-1)(a-1)}$$

$$= \frac{(2a-1)(3a+2)}{(2a-1)(a-1)}$$

$$= \boxed{\frac{3a+2}{a-1}}$$

b. (5 points) $\frac{-15x^6y^4}{5x^4y}$

$$= \frac{-5 \cdot 3 \cdot x^6 y^4}{5 \cdot x^4 y}$$

$$= \boxed{-3x^2y^3}$$

2. (12 points) Solve the equation.

$$\frac{3x-1}{x-2} = \frac{5}{x-2} + 1$$

$$\frac{(x-2)}{1} \cdot \frac{(3x-1)}{(x-2)} = \frac{5}{(x-2)} \cdot \frac{(x-2)}{1} + \frac{1}{1} \cdot \frac{(x-2)}{1}$$

$$3x-1 = 5 + (x-2)$$

$$3x-1 = 3+x$$

$$2x = 4$$

$$\boxed{x=2}$$

3. (12 points) Divide the rational expression. Write the answer in lowest terms.

$$\frac{4a^2+9a+2}{3a^2+11a+10} \div \frac{4a^2+17a+4}{3a^2+2a-5}$$

$$= \frac{4a^2+9a+2}{3a^2+11a+10} \cdot \frac{3a^2+2a-5}{4a^2+17a+4}$$

$$= \frac{4a^2+8a+a+2}{3a^2+6a+5a+10} \cdot \frac{3a^2-3a+5a-5}{4a^2+16a+a+4}$$

$$= \frac{4a(a+2)+(a+2)}{3a(a+2)+5(a+2)} \cdot \frac{3a(a-1)+5(a-1)}{4a(a+4)+(a+4)}$$

$$= \frac{(a+2)(4a+1)}{(a+2)(3a+5)} \cdot \frac{(a-1)(3a+5)}{(a+4)(4a+1)}$$

$$= \boxed{\frac{a-1}{a+4}}$$

4. (10 points) Subtract the rational expression. Write the answer in lowest terms.

$$\frac{3}{2m^2-9m-5} - \frac{m+1}{2m^2-m-1}$$

Find LCD: $2m^2-9m-5 = 2m^2+m-10m-5 = (2m+1)(m-5)$

$2m^2-m-1 = 2m^2-2m+m-1 = (m-1)(2m+1)$

LCD $= (2m+1)(m-1)(m-5)$

$$\frac{3}{(2m+1)(m-5)} \cdot \frac{(m-1)}{(m-1)} - \frac{(m+1)}{(2m+1)(m-1)} \cdot \frac{(m-5)}{(m-5)}$$

$$= \frac{3m-3-(m+1)(m-5)}{(2m+1)(m-5)(m-1)}$$

$$= \frac{3m-3-[m^2-4m-5]}{(2m+1)(m-5)(m-1)} = \frac{3m-3-m^2+4m+5}{(2m+1)(m-5)(m-1)} = \boxed{\frac{-(m^2-7m-2)}{(2m+1)(m-5)(m-1)}}$$

5. (12 points) Simplify the complex fraction. Write the answer in lowest terms.

$$\frac{\left(\frac{1}{x+3}-1\right)}{\left(1+\frac{1}{x+3}\right)}$$

Common denom. is $x+3$

$$\frac{\frac{(x+3)}{1}\cdot\frac{1}{x+3}-\frac{(x+3)}{1}\cdot 1}{\frac{(x+3)}{1}\cdot 1+\frac{(x+3)}{1}\cdot\frac{1}{(x+3)}} = \frac{1-(x+3)}{(x+3)+1} = \boxed{\frac{-x-2}{x+4}}$$

6. (10 points) Rationalize the denominators.

a. (6 points) $\frac{-3}{4-\sqrt{3}}$

$$= \frac{-3}{4-\sqrt{3}}\cdot\frac{(4+\sqrt{3})}{(4+\sqrt{3})}$$

$$= \frac{-12-3\sqrt{3}}{16-(\sqrt{3})^2}$$

$$= \boxed{\frac{-12-3\sqrt{3}}{13}}$$

b. (4 points) $\sqrt{\frac{2}{3x}}$

$$= \frac{\sqrt{2}}{\sqrt{3x}}\cdot\frac{\sqrt{3x}}{\sqrt{3x}}$$

$$= \boxed{\frac{\sqrt{6x}}{3x}}$$

7. (14 points) Simplify the expressions.

   a. (4 points) $\dfrac{20\sqrt{18}}{5\sqrt{3}}$

   b. (4 points) $3\sqrt{28}+\sqrt{63}$

   c. (6 points) $(2-\sqrt{7})(3\sqrt{2}+1)$

a) $\frac{20}{5}\cdot\frac{\sqrt{18}}{\sqrt{3}} = 4\cdot\sqrt{\frac{18}{3}} = \boxed{4\cdot\sqrt{6}}$

b) $3\sqrt{4\cdot7}+\sqrt{9\cdot7} = 3\cdot2\sqrt{7}+3\sqrt{7} = 6\sqrt{7}+3\sqrt{7} = \boxed{9\sqrt{7}}$

c) $(2-\sqrt{7})(3\sqrt{2}+1) = 2\cdot3\sqrt{7}+2\cdot1+(-\sqrt{7})(3\sqrt{2})+(-\sqrt{7})(1)$

$= 6\sqrt{7}+2-3\sqrt{14}-\sqrt{7}$

$= \boxed{5\sqrt{7}-3\sqrt{14}+2}$

8. (10 points) A boat goes 7 mph in still water. It takes as long to go 20 mi upstream as 50 mi downstream. Find the speed of the current.

means times are equal

x = speed of current $\quad d = r\cdot t$

	rate	distance	time
upstream	$7-x$	20	$\frac{20}{7-x}$
downstream	$7+x$	50	$\frac{50}{7+x}$

$\frac{20}{7-x} = \frac{50}{7+x}$ $\quad 20(7+x) = 50(7-x)$

$140+20x = 350-50x$

$70x = 210$

$\boxed{x = 3}$

Extra Credit (10 points) Simplify the expression.

$$r+\frac{r}{4-\left(\frac{2}{6+2}\right)}$$

$\rightarrow 4-\frac{2}{6+2} = 4-\frac{2}{8} = 4-\frac{1}{4} = \frac{16-1}{4} = \frac{15}{4}$

$\rightarrow r+\frac{r}{\frac{15}{4}} = r+r\cdot\frac{4}{15}$

$= r+\frac{4r}{15}$

$= \frac{15r}{15}+\frac{4r}{15}$

$= \boxed{\frac{19r}{15}}$