

①  $70 = 2 \cdot 7 \cdot 5$   
 $126 = 2 \cdot 3 \cdot 3 \cdot 7$   
 $42 = 2 \cdot 3 \cdot 7$   
 $56 = 2 \cdot 2 \cdot 2 \cdot 7$

GCF =  $2 \cdot 7 = 14$   
 $70 = 14 \cdot 5$   
 $126 = 14 \cdot 9$   
 $42 = 14 \cdot 3$   
 $56 = 14 \cdot 4$

②  $84 = 2 \cdot 2 \cdot 3 \cdot 7$   
 $280 = 2 \cdot 2 \cdot 2 \cdot 5 \cdot 7$   
 $112 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 7$

GCF =  $2 \cdot 2 \cdot 7 = 28$   
 $84 = 28 \cdot 3$   
 $280 = 28 \cdot 10$   
 $112 = 28 \cdot 4$

③  $y^7 z^2$   
 $y^4 z^8$   
 $z^3$

GCF =  $z^2$

$y^7 z^2 = z^2 \cdot y^7$   
 $y^4 z^8 = z^2 (y^4 z^6)$   
 $z^3 = z^2 (z)$

④  $45a^7y^4 = 3 \cdot 3 \cdot 5 a^7 y^4$   
 $75a^3y^2 = 3 \cdot 5 \cdot 5 a^3 y^2$   
 $90a^2y = 2 \cdot 3 \cdot 3 \cdot 5 a^2 y$   
 $30a^4y^3 = 2 \cdot 3 \cdot 5 a^4 y^3$

GCF =  $45a^7y^4 = 15a^2y \cdot (3a^5y^3)$   
 $75a^3y^2 = 15a^2y \cdot (5y)$   
 $90a^2y = 15a^2y \cdot (6)$   
 $30a^4y^3 = 15a^2y \cdot (2a^2y^2)$   
 $3 \cdot 5 \cdot a^2 \cdot y = 15a^2y$

⑤  $84 = 4(21)$

⑥  $-75a^4y^2 = 25a^2y^2(-3a)$

⑦ Find GCF

$45xy = 3 \cdot 3 \cdot 5xy$   
 $18x = 3 \cdot 3 \cdot 2x$   
 $27x^3y = 3 \cdot 3 \cdot 3x^3y$

GCF =  $3 \cdot 3x = 9x$

$9x[5y + 2 + 3x^2y]$

⑧ Find GCF

$56x^2y^4 = 2 \cdot 2 \cdot 2 \cdot 7x^2y^4$   
 $-24xy^3 = -2 \cdot 2 \cdot 2 \cdot 3xy^3$   
 $32xy^2 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2xy^2$

GCF =  $2 \cdot 2 \cdot 2xy^2 = 8xy^2$

$56x^2y^4 - 24xy^3 + 32xy^2$   
 $= 8xy^2[7xy^2 - 3y + 4]$

9)  $x^4 + 2x^2 + 5x^2 + 10$   
 group 1      group 2  
 $x^4$  } GCF =  $x^2$      $5x^2$  } GCF = 5  
 $2x^2$  }  
 $= x^2(x^2 + 2) + 5(x^2 + 2)$   
 $= (x^2 + 2)(x^2 + 5)$

10)  $12x^2 + 4xy - 6xy - 2y^2$   
 group 1      group 2  
 $12x^2$  } GCF =  $4x$      $-6xy$  }  $-2y$   
 $4xy$  }                     $-2y^2$  }  
 $= 4x(3x + y) + (-2y)(3x + y)$   
 $= (3x + y)(4x - 2y)$

11) Factors of 28  
 2, 14    -2, 14  
 4, 7    -4, -7  
 The pairs that sums to -11  
 $-4, -7$

12) Factors of -54  
 -2, 27    2, -27  
 -3, 18    3, -18  
 -6, 9    6, -9  
 The pair that sums to -3  
 $6, -9$

13) Need to pick a number so  
 $4 \cdot ( ) = -28$   
 $4 + ( ) = -11$   
 $\rightarrow -7$   
 $x^2 + 3x - 28 = (x - 4)(x - 7)$

14)  $x^2 - x - 30 = (x + 5)( )$   
 find number so  $5( ) = -30$   
 $5 + ( ) = -1$   
 $\rightarrow -6$   
 $x^2 - x - 30 = (x + 5)(x - 6)$

15)  $x^2 - 11x + 28$   
 Factors of 28: 4, 7    -4, -7  
                   2, 14    -2, -14  
 The pair that sum to -11:  $-4, -7$

16)  $x^2 - 8x - 33$   
 Factors of -33: -1, 33    -33, 1  
 the pair that sum to -8: None  
 the polynomial is prime

17)  $3p^6 + 18p^5 + 24p^4$   
 Find a GCF if any  
 $3p^6 = 3p^6$   
 $18p^5 = 2 \cdot 3 \cdot 3p^5$  } GCF =  $3p^4$   
 $24p^4 = 2 \cdot 2 \cdot 2 \cdot 3p^4$

$3p^6 + 18p^5 + 24p^4 = 3p^4(p^2 + 6p + 8)$   
 $\rightarrow$  we can <sup>try to</sup> factor the quadratic  
 factors of 8: 2, 4    ~~-2, -4~~  
 (only need positive factors b/c b=6 is positive)

$3p^4(p^2 + 6p + 8) = 3p^4(p + 4)(p + 2)$

Worksheet 6

(18)  $3x^2y^2 - 24xy + 36x$

Find GCF if any

$$\left. \begin{aligned} 3x^2y^2 &= 3x^2y^2 \\ -24xy &= -2 \cdot 2 \cdot 2 \cdot 3xy \\ 36x &= 2 \cdot 2 \cdot 3 \cdot 3x \end{aligned} \right\} 3x$$

$3x(y^2 - 8y + 12)$

check if quadratic is factorable

negative factors of 12:

- 2, -6
- 3, -4

The pair that sum to -8

- 2, -6

$3x(y-2)(y-6)$

(19)  $3x^2 + 13x + 14$

step 1  $3 \cdot 14 = 42$

step 2 factors of 42 that are positive

- 2, 21
- 3, 14

6, 7 ← these sum to 13

step 3:  $13x = 6x + 7x$

$$\underbrace{3x^2 + 6x}_{\text{group 1}} + \underbrace{7x + 14}_{\text{group 2}}$$

$= 3x(x+2) + 7(x+2)$

$= (x+2)(3x+7)$

(3)

(20)  $25a^2 + 30ab + 9b^2$

$a \cdot c = 25 \cdot 9 = 225$

→ two factors of 225 that add to 30  
→ 15, 15

$30ab = 15ab + 15ab$

$25a^2 + 15ab + 15ab + 9b^2$

$= 5a(5a + 3b) + 3b(5a + 3b)$

$= (5a + 3b)(5a + 3b)$

$= (5a + 3b)^2$

(21)  $6x^2 + 19x + 10 = (3x+2)(\quad)$

$6 \cdot 10 = 60$

→ use 4, 15 →  $4 + 15 = 19$

$4 \cdot 15 = 60$

→  $19x = 4x + 15x$

$6x^2 + 4x + 15x + 10$

$= 2x(3x+2) + 5(3x+2)$

$= (3x+2)(2x+5)$

(22)  $24y^2 - 17y + 3 = (3y-1)(\quad)$

→  $a \cdot c = 24 \cdot 3 = 72$

→ factors of 72 whose sum is -17: -9, -8

→  $-17y = -8y - 9y$

$24y^2 - 8y - 9y + 3$

$= 8y(3y-1) - 3(3y-1)$

$= (3y-1)(8y-3)$

$$(23) \quad 4y^2 + 3y - 10$$

I'm going to use grouping method

$$\rightarrow 4 \cdot (-10) = -40$$

$$\rightarrow \text{factors: } -5, 8$$

$$3y = 8y + (-5)y$$

$$4y^2 + 8y - 5y - 10$$

$$= 4y(y+2) + (-5)(y+2)$$

$$= \boxed{(y+2)(4y-5)}$$

$$(24) \quad 14b^2 + 3b - 2$$

$$\rightarrow a \cdot c = 14(-2) = -28$$

$$\rightarrow \text{factors: } 7, -4$$

$$3b = 7b + (-4b)$$

$$14b^2 + 7b - 4b - 2$$

$$= 7b(2b+1) + (-2)(2b+1)$$

$$= \boxed{(2b+1)(7b-2)}$$

$$(25) \quad \text{A difference of squares:}$$

$$(x^2 - 49) = (x+7)(x-7)$$

or factors of -49

$$-7, 7$$

sum is 0

$$0x = 7x - 7x$$

$$x^2 + 7x - 7x - 49 = x(x+7) + (-7)(x+7)$$

$$(26) \quad 100r^2 - 9s^2 = (10r - 3s)(10r + 3s)$$

$$\text{or } 100(-9) = -900$$

Two factors whose sum is 0: 30, -30

$$100r^2 + 30rs - 30rs - 9s^2$$

$$= 10r(10r + 3s) + (-3s)(10r + 3s)$$

$$= \boxed{(10r + 3s)(10r - 3s)}$$

$$(27) \quad m^2 - 8m + 16$$

factors of 16 whose sum is -8: -4, -4

$$-8m = -4m + -4m$$

$$m^2 - 4m - 4m + 16 = m(m-4) + (-4)(m-4)$$

$$= (m-4)(m-4) = \boxed{(m-4)^2}$$

$$(28) \quad -16x^2 - 48x - 36$$

$$= -4(4x^2 + 12x + 9)$$

$$(4)(9) = 36 \rightarrow \text{factors } 6, 6$$

$$= -4(4x^2 + 6x + 6x + 9)$$

$$= -4 [2x(2x+3) + 3(2x+3)]$$

$$= -4 [(2x+3)(2x+3)] = \boxed{-4(2x+3)^2}$$

$$(29) \quad b^3 - 27$$

use the special form

$$= \boxed{(b-3)(b^2 + 3b + 9)}$$

$$(30) \quad c^3 - 8 = \boxed{(c-2)(c^2 + 2c + 4)}$$

$$(31) \quad m^3 + 64 = \boxed{(m+4)(m^2 - 4m + 16)}$$

$$(32) \quad y^3 + 27 = (y+3)(y^2 - 3y + 9)$$

(33)  $2x^2 - 3x - 20 = 0$

factor

$\rightarrow 2(-20) = -40$

$\rightarrow$  factors  $-8, 5$

$-3x = -8x + 5x$

$2x^2 - 8x + 5x - 20$

$= 2x(x-4) + 5(x-4)$

$= (x-4)(2x+5)$

~~$(x-4)(2x+5) = 0$~~

$x-4 = 0 \quad 2x+5 = 0$

$x = 4$

$2x = -5$

$x = -5/2$

(34)  $12x^2 + 7x - 12 = 0$

$\rightarrow 12(-12) = -144$

~~$\rightarrow 9, 16$~~

$-9, 16$

$7x = -9x + 16x$

$12x^2 - 9x + 16x - 12 = 0$

$3x(4x-3) + 4(4x-3) = 0$

$(4x-3)(3x+4) = 0$

$4x-3 = 0$

$3x+4 = 0$

$4x = 3$

$3x = -4$

$x = 3/4$

$x = -4/3$

(35)  $x(2x^2 - 7x - 15) = 0$

factor  $2x^2 - 7x - 15$

$2(15) = -30$

$\rightarrow -10, 3$

$2x^2 - 10x + 3x - 15$

$= 2x(x-5) + 3(x-5)$

$= (x-5)(2x+3)$

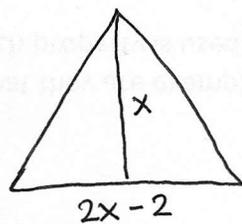
$\rightarrow x(x-5)(2x+3) = 0$

$x = 0 \quad x-5 = 0 \quad 2x+3 = 0$

$x = 5$

$2x = -3$

$x = -3/2$



x = height of triangle

$42 = \frac{1}{2}x(2x-2)$

$0 = \frac{1}{2}[2x^2 - 2x] - 42$

~~$0 = 2(x^2 - x - 42)$~~

$0 = x^2 - x - 42$

$0 = (x-7)(x+6)$

$x-7 = 0$

$x+6 = 0$

$x = 7$

$x = -6$

height must be positive

$x = 7$  height

$2(7) - 2 = 12$  base.

(36)  $x^3 + 2x^2 - 8x = 0$

$x(x^2 + 2x - 8) = 0$

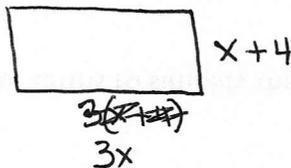
$x(x+4)(x-2) = 0$

$x = 0 \quad x+4 = 0 \quad x-2 = 0$

$x = 0 \quad x = -4$

$x = 2$

(37)



x = original width

$3x(x+4) = 231$

$3x^2 + 12x - 231 = 0$

$3(x^2 + 4x - 77) = 0$

$3(x+11)(x-7) = 0$

$(x+11)(x-7) = 0$

$(x+11) = 0 \quad x-7 = 0$

$x = -11$

$x = 7$

width can't be neg.

width = 7

length =  $3(7) = 21$

factors of -77 that add to 4  
 $-7, 11$

$x^2 + 11x - 7x - 77$

$= (x)(x+11) + (-7)(x+11)$

$= (x+11)(x-7)$

# Worksheet 6

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Two consecutive even integers  $x, x+2$

$$x(x+2) = 24 + 3(x+2)$$

$$x^2 + 2x = 24 + 3x + 6$$

$$x^2 + 2x - 3x - 30 = 0$$

$$x^2 - x - 30 = 0$$

$$(x-6)(x+5) = 0$$

$$x-6=0 \quad x+5=0$$

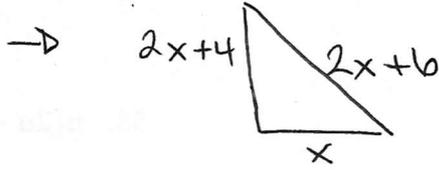
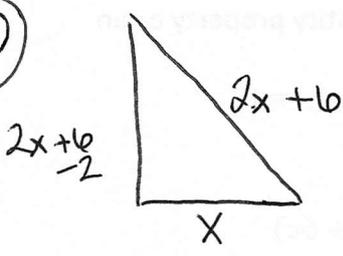
$$x=6 \quad x=-5$$

we want even integers

$$\boxed{x=6}$$
  
$$\boxed{x+2=8}$$

40

$x$  = Shortest side of triangle



Use pythagorean theorem

$$a^2 + b^2 = c^2$$

$$(2x+4)^2 + (x)^2 = (2x+6)^2$$

$$4x^2 + 16x + 16 + x^2 = 4x^2 + 24x + 36$$

$$5x^2 - 4x^2 + 16x - 24x + 16 - 36 = 0$$

$$x^2 - 8x - 20 = 0$$

$$(x-10)(x+2) = 0$$

$$x-10=0 \quad x+2=0$$
  
$$\boxed{x=10} \quad \boxed{x=-2}$$

length should be positive

short ~~side~~<sup>leg</sup>:  $x=10$

long leg:  $2x+4=24$

hypotenuse:  $2x+6=26$

-20

~~10~~

$\rightarrow -10, 2$

$$x^2 + 2x - 10x - 20$$

$$= x(x+2) + (-10)(x+2)$$

$$= (x+2)(x-10)$$