

Exam 4

PLEASE READ ALL THE DIRECTIONS CAREFULLY

- ❖ Show all work. Solutions without proper work will receive no credit.
- ❖ Present work in a clear, organized manner.
- ❖ No notes, books, or calculators allowed.
- ❖ Write answers in **lowest terms** when appropriate
- ❖ Good Luck!



Problem	1	2	3	4	5	6	7	Bonus	Total
Score									
Possible	18	25	9	14	26	8		10	100

1. (18 points) Divide the polynomials. Use long division when applicable.

a. (5 points) $\frac{8x^3y^2 - 9x^2y^3 + 5xy}{9xy^2}$

$$= \frac{8x^3y^2}{9xy^2} - \frac{9x^2y^3}{9xy^2} + \frac{5xy}{9xy^2}$$

Divide by a ~~no~~ monomial

$$= \boxed{\frac{8x^2}{9} - \frac{xy}{1} + \frac{5}{9y}}$$

b. (6 points) $\frac{6x^2 + x - 6}{2x - 1}$

← long division

$$\begin{array}{r} 3x+2 \text{ R}-4 \\ 2x-1 \overline{) 6x^2+x-6} \\ \underline{-(6x^2-3x)} \\ 4x-6 \\ \underline{-(4x-2)} \\ -4 \end{array}$$

$$\text{Ans: } \boxed{3x+2 - \frac{4}{2x-1}}$$

c. (7 points) $\frac{8x^3+27}{x+4}$ long division

$$\begin{array}{r} 8x^2 - 32x + 128 \text{ R}-485 \\ x+4 \overline{) 8x^3 + 0x^2 + 0x + 27} \\ \underline{-(8x^3 + 32x^2)} \\ -32x^2 + 0x \\ \underline{-(-32x^2 - 128x)} \\ 128x + 27 \\ \underline{-(128x + 512)} \\ -485 \end{array}$$

~~2~~

$$\text{Ans: } \boxed{8x^2 - 32x + 128 - \frac{485}{x+4}}$$

2. (25 points) Factor the expressions.

a. (5 points) $ad^2 + 10ad + 25a^2$

$$= a(d^2 + 10d + 25a)$$

not factorable

b. (5 points) $15x^4y^2 + 24x^6y^6 - 32x^7y^3$

$$= x^4 y^2 (15 + 24x^2 y^4 - 32x^3 y)$$

c. (7 points) $4p^4 + 36p^3 + 32p^2$

$$= 4p^2(p^2 + 9p + 8)$$
$$= \boxed{4p^2(p+8)(p+1)}$$

p: 8
s: 9

d. (8 points) $-2y^3 - 3y^2 - y$

$$= -y(2y^2 + 3y + 1)$$

$$= -y(2y^2 + 2y + y + 1)$$

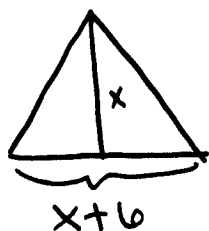
$$= -y[2y(y+1) + 1(y+1)]$$

$$= \boxed{-y(y+1)(2y+1)}$$

p: 2 } 1, 2
s: 3 }

3. (9 points) The base of a triangle is 6 inches greater than the height. The area is 20 square inches.

- (3 points) Draw a diagram from the information provided.
- (2 points) State the equation for the problem.
- (4 points) Find the length of the triangle's base and height.



$$\text{Area} = 20 \rightarrow A = \frac{1}{2}b \cdot h$$

$$20 = \frac{1}{2}(x+6)x$$

$$40 = x^2 + 6x$$

$$0 = x^2 + 6x - 40$$

$$0 = (x+10)(x-4)$$

$$x+10=0$$

$$x = -10$$

$$x-4=0$$

$$x = 4$$

$$\begin{matrix} p: -40 \\ s: 6 \end{matrix} \left. \vphantom{\begin{matrix} p: -40 \\ s: 6 \end{matrix}} \right\} 10, -4$$

Height: $x = 4$ inches
Base: $x+6 = 10$ inches

choose positive length

4. (14 points) Solve the equation for x

$$\frac{4}{x} - \frac{2}{x+1} = \frac{4}{3}$$

$$\text{LCD: } x(x+1)(3) = 3x(x+1)$$

Multiply by LCD

$$\frac{4 \cdot (3x)(x+1)}{x} - \frac{2 \cdot (3x)(x+1)}{(x+1)} = \frac{4 \cdot (3x)(x+1)}{3}$$

$$4 \cdot 3(x+1) - 2 \cdot 3x = 4(x)(x+1)$$

$$12x + 12 - 6x = 4x^2 + 4x$$

$$6x + 12 = 4x^2 + 4x$$

$$-6x - 12 - 6x - 12$$

$$0 = 4x^2 - 2x - 12$$

$$0 = 4x^2 - 2x - 12$$

$$0 = 2(2x^2 - x - 6)$$

$$0 = 2x^2 - x - 6$$

$$\begin{matrix} p: -12 \\ s: -1 \end{matrix} \left. \vphantom{\begin{matrix} p: -12 \\ s: -1 \end{matrix}} \right\} -4, 3$$

$$\begin{aligned} 0 &= 2x^2 - 4x + 3x - 6 \\ 0 &= 2x(x-2) + 3(x-2) \\ 0 &= (x-2)(2x+3) \end{aligned}$$

$$\begin{matrix} x-2=0 & 2x+3=0 \\ \boxed{x=2} & \boxed{x=-3/2} \end{matrix}$$

Check answers or state restrictions!

$$\begin{matrix} x+1 \neq 0 \\ \boxed{x \neq -1} \end{matrix}$$

5. (26 points) Perform the indicated operation (multiply, divide, add, or subtract) for the rational expressions and write in lowest terms.

a. (5 points) $\frac{x^2-8x+15}{x^2-9x+4} \cdot \frac{7-x}{x^2+4x-21}$

$$\frac{(x-5)(x-3) \cdot (7-x)}{(x^2-9x+4)(x+7)(x+3)} = \frac{(x-5)(7-x)}{(x^2-9x+4)(x+7)}$$

1) p: 18 { 9, 2
s: 11 }
 $3x^2+9x+2x+6$
 $= 3x(x+3)+2(x+3)$
 $= (3x+2)(x+3)$

2) p: 28 { 14, 2
s: 16 }
 $4x^2+2x+14x+7$
 $2x(2x+1)+7(2x+1)$
 $(2x+7)(2x+1)$

b. (8 points) $\frac{3x^2+11x+6}{4x^2+16x+7} \div \frac{9x^2+12x+4}{2x^2-x-28}$

3) p: 36 { 6, 6
s: 12 }
 $(3x+2)^2$

$$\frac{(3x+2)(x+3)}{(2x+7)(2x+1)} \cdot \frac{(2x+7)(x-4)}{(3x+2)(3x+2)}$$

$$= \frac{(x+3)(x-4)}{(2x+1)(3x+2)}$$

4) p: -56 { 7, -8
s: -1 }

$$2x^2-8x+7-28$$

 $2x(x-4)+7(x-4)$
 $(2x+7)(x-4)$

c. (5 points) $\frac{2x+1}{x^2-x-6} - \frac{x-1}{x^2-x-6}$

$$= \frac{2x-x+1+1}{x^2-x-6} = \frac{x+2}{(x-3)(x+2)} = \frac{1}{x-3}$$

d. (8 points) $\frac{2x}{x^2+x-12} + \frac{3x}{x^2-9}$

$$= \frac{2x}{(x+4)(x-3)} + \frac{3x}{(x+3)(x-3)}$$

$$= \frac{2x(x+3)}{(x+4)(x^2-9)} + \frac{3x(x+4)}{(x+4)(x^2-9)}$$

$$= \frac{2x^2+6x+3x^2+12x}{(x+4)(x^2-9)}$$

$$= \frac{5x^2+18x}{(x+4)(x^2-9)}$$

6. (8 points) Simplify the complex fraction.

$$\frac{\frac{x^2-9}{x} \cdot \frac{1}{x-3}}{\frac{\frac{x^2-9}{x}}{x-3}} = \frac{(x-3)(x+3)}{(x)(x-3)}$$

$$\boxed{= \frac{x+3}{x}}$$

Bonus: Simplify the complex fraction.

$$\text{LCD: } x^2 \cdot y$$

$$\frac{\frac{\frac{x^2 y}{x} - \frac{x^2 y}{y}}{\frac{y \cdot x^2 y}{x^2} - \frac{x^2 y}{y}}}{\frac{\frac{1}{x} - \frac{1}{y}}{\frac{y}{x^2} - \frac{1}{y}}} = \frac{xy - x^2}{y^2 - x^2} = \frac{x(y-x)}{(y+x)(y-x)} = \boxed{\frac{x}{y+x}}$$