Math 100
American River College

1. (10 points) Simplify the radical expressions

a. (3 points) $\sqrt{\frac{75x^8}{81y^{12}}}$

$= \frac{\sqrt{75x^8}}{\sqrt{81y^{12}}}$

$= \frac{\sqrt{25}\sqrt{x^8}\sqrt{3}}{\sqrt{81}\sqrt{y^{12}}} = \boxed{\frac{5x^4\sqrt{3}}{9y^6}}$

b. (3 points) $\sqrt[3]{64x^9y^2}$

$= \sqrt[3]{4^3x^9}\sqrt[3]{y^2}$

$= \boxed{4x^3\sqrt[3]{y^2}}$

c. (4 points) $\frac{12-\sqrt{18}}{21}$

$= \frac{3\cdot 4 - \sqrt{9}\sqrt{2}}{3\cdot 7}$

$= \frac{3\cdot 4 - 3\sqrt{2}}{3\cdot 7}$

$= \frac{3(4-\sqrt{2})}{3}$

$= \boxed{4-\sqrt{2}}$

2. (10 points) Solve the variation problems.

a. y varies directly as x and $y = 4$ when $x = 20$.
   i. State the variation equation.
   ii. Find the constant of variation k
   iii. Find the value of x when $y = 5$.

i) $y = kx$

ii) $4 = k(20)$

$\boxed{k = 1/5}$

iii) $y = kx$

$5 = \frac{1}{5}\cdot x$

$\boxed{25 = x}$

b. y varies inversely as x and $y = 8$ when $x = 10$.
   i. State the variation equation.
   ii. Find the constant of variation k
   iii. Find the value of y when $x = 20$.

i) $y = \frac{k}{x}$

ii) $8 = \frac{k}{10}$

$\boxed{80 = k}$

iii) $y = \frac{80}{x}$

$y = \frac{80}{20}$

$\boxed{y = 4}$

Math 100
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3. (12 points) Rationalize the denominators.

a. (5 points) $\frac{\sqrt{45y}}{\sqrt{5y}}$

$= \sqrt{\frac{45y}{5y}}$

$= \sqrt{9}$

$= \boxed{3}$

b. (7 points) $\frac{2}{\sqrt{6}-2} \cdot \frac{\sqrt{6}+2}{\sqrt{6}+2}$

$= \frac{2(\sqrt{6}+2)}{6-4}$

$= \frac{2(\sqrt{6}+2)}{2}$

$= \boxed{\sqrt{6}+2}$

4. (12 points) Simplify the expressions. Write answers in exponential form and with positive exponents.

a. (3 points) $4^{\frac{3}{2}} = \left(4^{1/2}\right)^3 = \left(\sqrt{4}\right)^3 = \boxed{(2)^3 = 8}$

b. (3 points) $\frac{2^{2/3}}{2^{-1/3}} = 2^{2/3} \cdot 2^{1/3} = \boxed{2^1}$

c. (6 points) $\left(2x^{1/2}\right)^3 \cdot \left(3x^{1/3}\right)^2$

$= 2^3 \cdot x^{3/2} \cdot 3^2 \cdot x^{2/3}$

$= 2^3 \cdot 3^2 \cdot x^{3/2} \cdot x^{2/3}$

$= 8 \cdot 9 \, x^{13/6}$

$= \boxed{72 x^{13/6}}$

Add the exponents

& $\frac{3}{2} + \frac{2}{3}$

$\frac{3}{2} \cdot \frac{3}{3} + \frac{2}{3} \cdot \frac{2}{2}$

$\frac{9}{6} + \frac{4}{6} = \frac{13}{6}$

5. (20 points) Simplify the radical expressions.

a. (3 points) $\sqrt{a} + 4\sqrt{a} - 2\sqrt{a}$

$= \boxed{3\sqrt{a}}$

b. (5 points) $\sqrt{36x^3} + \sqrt{81x^3}$

$= \sqrt{36}\sqrt{x^2}\sqrt{x} + \sqrt{81}\sqrt{x^2}\sqrt{x}$

$= 6x\sqrt{x} + 9x\sqrt{x}$

$= \boxed{15x\sqrt{x}}$

c. (5 points) $\sqrt{3}(\sqrt{2} + 2\sqrt{12})$

$= \sqrt{3}\sqrt{2} + 2\sqrt{3}\sqrt{12}$

$= \sqrt{6} + 2\sqrt{36}$

$= \sqrt{6} + 2 \cdot 6$

$= \boxed{\sqrt{6} + 12}$

d. (7 points) $(4\sqrt{3} + \sqrt{2})(\sqrt{3} - 2\sqrt{2})$

$= 4\sqrt{3}\sqrt{3} - 4 \cdot 2\sqrt{3}\sqrt{2} + \sqrt{2}\sqrt{3} - 2\sqrt{2}\sqrt{2}$

$= 4 \cdot 3 - 8\sqrt{6} + \sqrt{6} - 2 \cdot 2$

$= 12 - 4 - 7\sqrt{6}$

$= \boxed{8 - 7\sqrt{6}}$

Math 100
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6. (20 points) Solve each equation for x.

a. (3 points) $(x-5)^2 = 36$

$\sqrt{(x-5)^2} = \pm\sqrt{36}$

$x-5 = \pm 6$

$x = 5 \pm 6$

$x = 5+6 \quad x = 5-6$

$\boxed{x = 11 \quad x = -1}$

b. (3 points) $(x+2)^2 = -25$

$\sqrt{(x+2)^2} = \pm\sqrt{-25}$ ← Not a Real Sol'n

$x+2 = \pm 5i$ ← or

$\boxed{x = -2 \pm 5i}$

c. (6 points) Solve this equation by using the quadratic equation.

$2x^2 + 5x - 3 = 0 \qquad a=2 \quad b=5 \quad c=-3$

$x = \dfrac{-5 \pm \sqrt{25-4(2)(-3)}}{2(2)}$

$x = \dfrac{-5 \pm \sqrt{25+24}}{4}$

$x = \dfrac{-5 \pm \sqrt{49}}{4}$

$x = \dfrac{-5 \pm 7}{4}$

$x = \dfrac{-5+7}{4} \quad x = \dfrac{-5-7}{4}$

$x = \dfrac{2}{4} \quad x = \dfrac{-12}{4}$

$\boxed{x = 1/2} \quad \boxed{x = -3}$

d. (8 points) Solve this equation by using the completing the square.

$x^2 + 6x - 7 = 0$

$x^2+6x = 7$

$\left(\frac{b}{2}\right)^2 = (3)^2$

$x^2+6x+9 = 7+9$

$(x+3)^2 = 16$

$\sqrt{(x+3)^2} = \pm\sqrt{16}$

$x+3 = \pm 4$

$x = -3 \pm 4$

$x = -3+4 \quad x = -3-4$

$\boxed{x = 1} \quad \boxed{x = -7}$

Math 100
American River College

7. (16 points) Solve the radical equations.

a. (6 points) $\sqrt{6-x}=3$

$(\sqrt{6-x})^2 = 3^2$

$6-x=9$

$x=-3$

Check

$\sqrt{6+3}=3$

$\sqrt{9}=3$

$3=3$ ✓ True

$x=-3$ is the soln

b. (10 points) $x+1=\sqrt{x+7}$

$(x+1)^2 = (\sqrt{x+7})^2$

$x^2+2x+1 = x+7$

$x^2+x-6=0$

$(x+3)(x-2)=0$

$x+3=0$ $x-2=0$

$x=-3$ $x=2$

Check the solns

$2+1=\sqrt{2+7}$

$3=\sqrt{9}$

$3=3$ ✓ True } $x=2$

$-3+1=\sqrt{-3+7}$

$-2=\sqrt{4}$

$-2=2$ False! } $x=-3$

$x=2$ is the solution

Bonus: Solve the equation for x by using any method (factoring, completing the square, or quadratic formula)

$(x-2)(5x+4)=3x^2-15x-12$

→ You only need to use one method. I'm showing all three.

$5x^2+4x-10x-8 = 3x^2-15x-12$

$2x^2+9x+4=0$

FACTORING

$p: 8$ $s: 9$ } 8, 1

$2x^2+8x+x+4=0$

$2x(x+4)+1(x+4)=0$

$(x+4)(2x+1)=0$

$x+4=0$ $2x+1=0$

$x=-4$ $x=-\frac{1}{2}$

COMPLETE THE SQUARE

$2x^2+\frac{9}{2}x+4=0$

$x^2+\frac{9}{2}x+2=0$

$\left(\frac{b}{2}\right)^2 = \left(\frac{9}{4}\right)^2$

$x^2+\frac{9}{2}x+\left(\frac{9}{4}\right)^2 = -2+\frac{81}{16}$

$\left(x+\frac{9}{4}\right)^2 = -\frac{32}{16}+\frac{81}{16}$

$\left(x+\frac{9}{4}\right)^2 = \frac{49}{16}$

$x+\frac{9}{4} = \pm\frac{7}{4}$

$x=-4$ $x=-\frac{1}{2}$

Quadratic Formula

$2x^2+9x+4=0$

$x=\frac{-9\pm\sqrt{81-4(2)(4)}}{2(2)}$

$x=\frac{-9\pm\sqrt{81-32}}{4}$

$x=\frac{-9\pm\sqrt{49}}{4}$

$x=\frac{-9\pm7}{4}$

$x=-\frac{1}{2}$

$x=-4$