Exam 1 Solutions

Problem 1

a) Slope: $\frac{3+1}{4-8} = \frac{4}{-4} = -1$

line: $y - 3 = -1(x-4)$

$y - 3 = -x + 4$

$\boxed{y = -x + 7}$

b) Slope: $2y + 5x = 6$

$2y = 6 - 5x$

$y = 3 - \frac{5}{2}x \rightarrow m = -\frac{5}{2}$ Same slope for parallel lines

line: $y - 0 = -\frac{5}{2}(x+7)$

$\boxed{y = -\frac{5}{2}x - \frac{35}{2}}$

c) Slope for $(4,3), (2,0)$

$m = \frac{3-0}{4-2} = \frac{3}{2}$

Slope for $(2,0), (-18,-12)$

$m = $ [redacted]

$\frac{0+12}{2+18} = \frac{12}{20} = \frac{3}{5}$

Slopes not the same, the points cannot be on the same line

(don't need to check the last slope)

Exam 1 Solutions

Problem 2

a) $\log_5(8x+4) = 2$

$5^{\log_5(8x+4)} = 5^2$

$8x+4 = 25$

$8x = 21$

$\boxed{x = \frac{21}{8}}$

b) $3^{x^2-5x-5} = \frac{1}{3}$

$3^{x^2-5x-5} = 3^{-1}$

$x^2-5x-5 = -1$

$x^2-5x-4 = 0$

$x = \frac{5 \pm \sqrt{25-4(1)(-4)}}{2}$

$= \frac{5 \pm \sqrt{25+16}}{2}$

$\boxed{x = \frac{5 \pm \sqrt{41}}{2}}$

Problem 3

a) $\sum xy = 1\cdot3 + 2\cdot3 + 4\cdot1 = 3+6+4 = \underline{13}$

$\sum x = 1+2+4 = \underline{7}$

$\sum y = 3+3+1 = \underline{7}$

$\sum x^2 = 1^2+2^2+4^2 = 1+4+16 = \underline{21}$

$m = \frac{3(13)-(7)(7)}{3(21)-(7)^2} = \frac{39-49}{63-49} = \frac{-10}{14} = -\frac{5}{7}$

$b = \frac{7+\frac{5}{7}(7)}{3} = \frac{7+5}{3} = \frac{12}{3} = 4$

$\boxed{y = -\frac{5}{7}x + 4}$

Problem 3 continued

b) The points have a strong negative correlation. Also, the least squares line has a negative slope (as seen in part a above)

c) $r=0$ is not bad for our least squares line since the line would be a horizontal line. Thus, the least squares line will fit the data exactly

Problem 4

a) V.A. when $x^2+3x-10=0$

$(x+5)(x-2)=0$

$\boxed{x=-5}$

$\boxed{x=2}$

b) H.A. holistic approach: given $x \to \infty$

$f(x) \approx \frac{x^2}{x^2} \to 1$ $\boxed{y=1}$

c) There are no holes because no factors are similar in denominator and numerator. (factors don't cancel)

d) $f(x) = \frac{(x-2)(x+3)}{(x-2)} \equiv x+3$

hole is at $y=2+3=5$

$\to \boxed{(2,5)}$

Exam 1 Solutions

Problem 5

a) $\log_3(x+6)^3 + \log_3(x-7) - \log_3(x+2)$

$= \log_3\left(\dfrac{(x+6)^3(x-7)}{(x+2)}\right)$

b) $\ln(x-3)^4 + \ln(x-2) - \ln\sqrt{8x+1}$

$= 4\ln(x-3) + \ln(x-2) - \frac{1}{2}\ln(8x+1)$

c) i) we have $x^2 - x - 6 \neq 0$

$(x-3)(x+2) \neq 0$

$x \neq 3 \quad x \neq -2$

domain: $(-\infty, -2) \cup (-2, 3) \cup (3, \infty)$

or all real numbers except $x = 3$, $x = -2$

c) ii) only restriction is

$x^2 - 9 \geq 0$

$x^2 \geq 9$

$|x| \geq \pm 3 \rightarrow x \geq 3$

$x \leq -3$

domain: $(-\infty, -3] \cup [3, \infty)$

Problem 6

a) $C(x) = R(x)$

$12x + 39 = 25x$

$39 = 13x$

$\boxed{x = 3}$

b) $P = R - C$

$P(x) = 25x - (12x + 39)$

$P(x) = 13x - 39$

$P(30) = 13(30) - 39$

$= 390 - 39$

$= 351$

c) $91 = 13x - 39$

$130 = 13x$

$\boxed{x = 10}$

Extra Credit

$\ln x = \frac{1}{2}\ln(7x - 10)$

$\ln x = \ln(7x-10)^{1/2}$

$x = (7x-10)^{1/2}$

$x^2 = 7x - 10$

$x^2 - 7x + 10 = 0$

$(x-5)(x-2) = 0$

$\boxed{x = 5,\ x = 2}$