Exam 1 Solutions

Problem 1

a) $\lim_{x \to -3} \frac{x^2+2x-3}{x+3} = \frac{0}{0}$ by direct substitution

$\to \lim_{x \to -3} \frac{(x+3)(x-1)}{(x+3)} = \lim_{x \to -3} x-1 = -3-1$

$= \boxed{-4}$

b) $\lim_{x \to \infty} \frac{x^2+2x-3}{4x^3-x+100} = \frac{\infty}{\infty}$ by direct substitution

$\to \lim_{x \to \infty} \frac{\frac{x^2}{x^3} + \frac{2x}{x^3} - \frac{3}{x^3}}{\frac{4x^3}{x^3} - \frac{x}{x^3} + \frac{100}{x^3}} = \lim_{x \to \infty} \frac{\frac{1}{x} + \frac{2}{x^2} - \frac{3}{x^3}}{4 - \frac{1}{x^2} + \frac{100}{x^3}}$

$= \frac{0}{4} = \boxed{0}$

c) $\lim_{x \to 0} \frac{\frac{1}{x+2} + \frac{1}{2}}{x} = \frac{\frac{1}{4}}{0}$ undefined value by direct substitution.

$\lim_{x \to 0^-} \frac{\frac{1}{x+2} + \frac{1}{2}}{x} = \frac{(+)}{(-)} \to -\infty$

$\lim_{x \to 0^+} \frac{\frac{1}{x+2} + \frac{1}{2}}{x} \quad \frac{(+)}{(+)} \to \infty$

The limit tends to $-\infty$ on the left and $+\infty$ on the right.

Thus the limit doesn't exist

Exam 2 Solutions

Problem 2

a) $$\frac{f(b)-f(a)}{b-a} = \frac{(-2)(-4+1)^2 - (2)(4-1)^2}{1+1}$$

$$= \frac{-2(9) - 2(9)}{2} = -18$$

b) $$f'(x) = 2(4x-x^3)^2 + 2x \cdot 2(4x-x^3)(4-3x^2)$$

c) Slope of tangent line @ $x=1$ is $f'(1)$

$$f'(1) = 2(4-1)^2 + 4(1)(4-1)(4-3)$$

$$= 18 + 12 = \underline{30}$$

A point on the graph: $(1, 18)$ from part a above or just recalculate

$$y - 18 = 30(x-1)$$

$$y = 30x - 30 + 18$$

$$\boxed{y = 30x - 12}$$

Exam 2 Solutions

Problem 3

$$\lim_{h\to 0} \frac{\sqrt{2(x+h)+1} - \sqrt{2x+1}}{h} \cdot \frac{\sqrt{2(x+h)+1} + \sqrt{2x+1}}{\sqrt{2(x+h)+1} + \sqrt{2x+1}}$$

$$= \lim_{h\to 0} \frac{(\cancel{2x}+2h+\cancel{1}) - (\cancel{2x}+\cancel{1})}{h\left(\sqrt{2(x+h)+1} + \sqrt{2x+1}\right)}$$

$$= \lim_{h\to 0} \frac{2\cancel{h}}{\cancel{h}\left(\sqrt{2(x+h)+1} + \sqrt{2x+1}\right)}$$

$$= \frac{2}{\sqrt{2x+1} + \sqrt{2x+1}} = \boxed{\frac{1}{\sqrt{2x+1}}}$$

Problem 4

a) $f'(x) = 2 \cdot \frac{1}{2} x^{-1/2} - 6(-\frac{1}{3}) \cdot x^{-4/3}$

b) $f'(x) = (6x^2+4) \cdot e^{2x^3+4x} \cdot \ln(7x) + \frac{7}{7x} \cdot e^{2x^3+4x}$

c) $f'(x) = \dfrac{(15x^4-8)(e^{2x}) - (2e^{2x})(3x^5-8x)}{(e^{2x})^2}$

Problem 5

a) $\lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} x^2 + x - 12 = 1 + 1 - 12 = \boxed{-10}$

$\lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} 3 - x = 3 - 1 = \boxed{2}$

The right & left limits do not agree.
Thus the limit does not exist at $x = 1$

b) We want $\lim_{x \to 1^-} f(x) = \lim_{x \to 1^+} f(x)$

$$\lim_{x \to 1^-} x^2 + x - 12 = \lim_{x \to 1^+} k(3 - x)$$

$$-10 = k(3-1)$$

$$\frac{-10}{2} = k$$

$$\boxed{k = -5}$$

Exam 2 Solutions

Problem 6

a) $A = Pe^{rt}$

$$30{,}000 = 10{,}000\, e^{0.08t}$$

$$3 = e^{0.08t}$$

$$\ln(3) = 0.08t \longrightarrow \boxed{t = \frac{\ln(3)}{0.08}}$$

b) $A = Pe^{rt}$

$$50{,}000 = 10{,}000\, e^{r \cdot 6}$$

$$5 = e^{r6}$$

$$\ln(5) = r \cdot 6$$

$$\boxed{r = \frac{\ln 5}{6}}$$

Extra Credit

A corner occurs at $x = 6$ (property of absolute value functions)

Moreover, the derivative does not exist <u>there</u> by the following

$$\lim_{h \to 0} \frac{|6+h-6| - |6-6|}{h} = \frac{0}{0} \text{ by direct sub}$$

$$\lim_{h \to 0^+} \frac{(6+h-6) - (6-6)}{h} = \lim_{h \to 0^+} \frac{h}{h} = 1$$

$$\lim_{h \to 0^-} \frac{-(6+h-6) + (6-6)}{h} = \lim_{h \to 0^-} \frac{-h}{h} = -1$$

using definition of absolute value fn ~~value~~ and definition of derivative at a point

Since limits don't agree at $x = 6$, the derivative doesn't exist.