No notes or calculators. Show all work.

Find the derivatives of the functions below.

1. (5 points)

$$f(x) = \frac{4x^3 - x^2 + 2x - 9}{\sqrt{x}}$$

There are two ways to solve this problem: (1) using only the power rule, and (2) using the quotient rule.

Power Rule only

$$\begin{aligned} f(x) &= \frac{4x^3}{x^{1/2}} - \frac{x^2}{x^{1/2}} + \frac{2x}{x^{1/2}} - \frac{9}{x^{1/2}} \\ &= 4x^{5/2} - x^{3/2} + 2x^{1/2} - 9x^{-1/2} \end{aligned}$$

so the derivative is

$$f'(x) = 4 \cdot \frac{5}{2}x^{3/2} - \frac{3}{2}x^{1/2} + 2 \cdot \frac{1}{2}x^{-1/2} - 9 \cdot \left(-\frac{1}{2}\right) x^{-3/2}$$

Quotient Rule

$$\begin{aligned} h(x) &= 4x^3 - x^2 + 2x - 9 & g(x) &= x^{1/2} \\ h'(x) &= 12x^2 - 2x + 2 & g'(x) &= \frac{1}{2}x^{-1/2} \end{aligned}$$

So the derivative is:

$$f'(x) = \frac{(12x^2 - 2x + 2) \cdot \left(x^{1/2}\right) - (4x^3 - x^2 + 2x - 9) \cdot \left(\frac{1}{2}x^{-1/2}\right)}{\left(\sqrt{x}\right)^2}$$

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2. (5 points)

$$f(x) = \left(12x^3 + \sqrt[3]{x} + 100\right)\left(8\sqrt{x} - 7x^2 + 5x - 9\right)$$

Must use the product rule . . . or you can FOIL out the right hand side (I advise against this).

$$\begin{aligned} h(x) &= 12x^3 + x^{1/3} + 100 & g(x) &= 8x^{1/2} - 7x^2 + 5x - 9 \\ h'(x) &= 36x^2 + \frac{1}{3}x^{-2/3} & g'(x) &= 4x^{-1/2} - 14x + 5 \end{aligned}$$

Plugging into the formula for the product rule:

$$f'(x) = \left(36x^2 + \frac{1}{3}x^{-2/3}\right) \cdot \left(8x^{1/2} - 7x^2 + 5x - 9\right) + \left(12x^3 + x^{1/3} + 100\right)\left(4x^{-1/2} - 14x + 5\right)$$