

1. § a) $x = \frac{y+1}{2y-1}$

$$x(2y-1) = y+1$$

$$2xy - x = y+1$$

$$2xy - y = 1+x$$

$$y(2x-1) = 1+x$$

$$y = \frac{1+x}{(2x-1)}$$

$$\downarrow$$

$$f^{-1}(x) = \frac{1+x}{(2x-1)}$$

d) $x = y^3 + 2$

$$y^3 = x-2$$

$$y = \sqrt[3]{x-2}$$

$$f^{-1}(x) = \sqrt[3]{x-2}$$

b) $x = \frac{1+e^y}{1-e^y}$

$$x(1-e^y) = 1+e^y$$

$$x - xe^y = 1+e^y$$

$$x-1 = e^y + xe^y$$

$$x-1 = e^y(1+x)$$

$$e^y = \frac{x-1}{x+1}$$

$$\ln(e^y) = \ln \left| \frac{x-1}{x+1} \right|$$

$$y = \ln \left| \frac{x-1}{x+1} \right|$$

$$f^{-1}(x) = \ln \left| \frac{x-1}{x+1} \right|$$

c) Typo

$$x = 2 + \ln|y|$$

$$x-2 = \ln|y|$$

$$e^{x-2} = e^{\ln|y|}$$

$$e^{x-2} = y$$

$$e^{x-2} = f^{-1}(x)$$

e) $x = 2^{10^y}$

$$\log_2 x = \log_2 2^{10^y}$$

$$\log_2 x = 10^y$$

$$\log_{10}(\log_2 x) = \log_{10}(10^y)$$

$$\log_{10}(\log_2 x) = y$$

$$f^{-1}(x) = \log_{10}(\log_2 x)$$

2a) $2\ln|x| + \frac{1}{2}\ln|x+1|$
 $= \ln|x^2| + \ln\sqrt{x+1}$
 $= \ln(x^2\sqrt{x+1})$

b) $\frac{1}{3} \left[2\ln(x+3) + \ln(x) + \ln(x^2-1) \right]$
 $= \frac{1}{3} \left[\ln(x+3)^2 + \ln \left(\frac{x}{x^2-1} \right) \right]$
 $= \frac{1}{3} \left[\ln \left(\frac{x(x+3)^2}{x^2-1} \right) \right]$
 $= \ln \left(\frac{x(x+3)^2}{x^2-1} \right)^{\frac{1}{3}}$

$$3a) \ln(\sqrt{x^2+1}) = \boxed{\frac{1}{2}\ln(x^2+1)}$$

$$\begin{aligned} b) \ln(x\sqrt[3]{x^2+1}) \\ &= \ln(x) + \ln(\sqrt[3]{x^2+1}) \\ &= \boxed{\ln(x) + \frac{1}{3}\ln(x^2+1)} \end{aligned}$$

$$\begin{aligned} c) \ln\left(\frac{3x(x+1)}{(2x+1)^2}\right) \\ &= \ln(3x(x+1)) - \ln(2x+1)^2 \\ &= \ln(3x) + \ln(x+1) - 2\ln(2x+1) \\ &= \boxed{\ln(3) + \ln(x) + \ln(x+1) - 2\ln(2x+1)} \end{aligned}$$

$$4a) 3^{x^2-5x-5} = 3^{-1}$$

$$x^2 - 5x - 5 = -1$$

$$x^2 - 5x - 4 = 0$$

$$x = \frac{5 \pm \sqrt{25 + 4(4)}}{2}$$

$$\boxed{x = \frac{5 \pm \sqrt{41}}{2}}$$

$$b) \ln(2x-1) = 3$$

$$e^{\ln(2x-1)} = e^3$$

$$2x-1 = e^3$$

$$\boxed{x = \frac{e^3 + 1}{2}}$$

$$c) e^{3x-4} = 2$$

$$\ln|e^{3x-4}| = \ln|2|$$

$$3x-4 = \ln|2|$$

$$\boxed{x = \frac{\ln|2| + 4}{3}}$$

$$d) \ln(x) - \frac{1}{2}\ln|4x+4| = 0$$

$$\ln|x| = \ln(4x+4)^{1/2}$$

$$x^2 = (4x+4)^{1/2}$$

$$x^2 = 4x+4$$

$$x^2 - 4x - 4 = 0$$

$$x = \frac{4 \pm \sqrt{16 + 4(4)}}{2} = \frac{4 \pm 4\sqrt{2}}{2} = \boxed{2 \pm 2\sqrt{2}}$$

Section 1.3 & 1.4

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4e) $e^{ax} = ce^{bx}$

$$e^{ax} = e^c e^{bx}$$

$$e^{ax} = e^{c+bx}$$

$$\ln e^{ax} = \ln e^{c+bx}$$

$$ax = c + bx$$

$$ax - bx = c$$

$$x(a-b) = c$$

$$x = \frac{c}{a-b}$$

5) a) $\sin^2 \theta - 1 = 0$

$$\sin^2 \theta = 1$$

$$\sin \theta = \pm 1$$

$$\sin \theta = 1 \quad \sin \theta = -1$$

$$\boxed{\theta = \pi/2} \quad \boxed{\theta = \frac{3\pi}{2}}$$

b) $\cos^2 \theta + \sin \theta = 1$

$$1 - \sin^2 \theta + \sin \theta = 1$$

$$\sin^2 \theta - \sin \theta = 0$$

$$\sin \theta (\sin \theta - 1) = 0$$

$$\sin \theta = 0 \quad \sin \theta = 1$$

$$\boxed{\theta = 0, \pi, 2\pi} \quad \boxed{\theta = \pi/2}$$

c) $2\cos^2 \theta - \cos \theta = 1$

$$2\cos^2 \theta - \cos \theta - 1 = 0$$

$$(2\cos \theta + 1)(\cos \theta - 1) = 0$$

$$\cos \theta = -\frac{1}{2} \quad \cos \theta = 1$$

$$\boxed{\theta = \frac{2\pi}{3}, \frac{4\pi}{3}} \quad \boxed{\theta = 0, 2\pi}$$