$\star$ means Advanced problem.

Section 2.2, 2.3, 2.4

1. Explain what is meant by the equation $\lim_{x\to 2} f(x) = 5$?

   Is it possible for your statement to be true and yet $f(2) = 3$? Explain.

2. Sketch a graph of an example function that satisfies the given conditions:

$$\lim_{x\to 3^+} f(x) = 4 \qquad \lim_{x\to 3^-} f(x) = 2 \qquad \lim_{x\to -2} f(x) = 2$$

3. Find the limit of each, if it exists

   (a) $\lim_{x\to -3} \frac{x^2-x+12}{x+3}$

   (b) $\lim_{t\to 0} \frac{\sqrt{2-t}-\sqrt{2}}{t}$

   (c) $\lim_{h\to 0} \frac{(h-5)^2-25}{h}$

   (d) $\lim_{x\to -3} \frac{x^2-x+12}{x+3}$

   (e) $\lim_{x\to -4} |x+4|$

   (f) $\lim_{x\to 1} \frac{x^2-1}{|x-1|}$

   (g) $\star \lim_{x\to 1} \left[\frac{1}{x-1} - \frac{2}{x^2-1}\right]$

4. Let

$$h(x) = \begin{cases} x & \text{if } x < 0 \\ x^2 & \text{if } 0 < x \le 2 \\ 8-x & \text{if } x > 2 \end{cases}$$

   Find each of the limits, if they exist

   (a) $\lim_{x\to 0^+} h(x)$

   (b) $\lim_{x\to 0} h(x)$

   (c) $\lim_{x\to 2^-} h(x)$

   (d) $\lim_{x\to 1} h(x)$

   (e) $\lim_{x\to 2^+} h(x)$

   (f) $\lim_{x\to 2} h(x)$

5. Find the limits

   (a) $\lim_{x\to 3} \frac{1}{(x-3)^8}$

   (b) $\lim_{h\to 0} \frac{x-1}{x^2(x+2)}$

6. $\star$ Use the Squeeze theorem to prove the limits

   (a) $\lim_{x\to 0} x^2 \cos(20\pi x) = 0$

   (b) $\star \lim_{x\to 0^+} \sqrt{x}e^{\sin(\pi/x)} = 0$