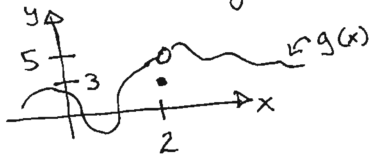
Handout 3 - Sections 2.2, 2.3, 2.4

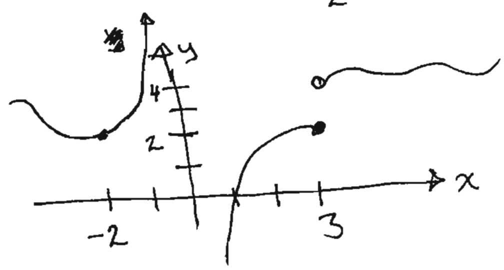
1. As x approaches 2, the function $f(x)$ gets arbitrarily close to 5. It is possible for the $\lim_{x \to 2} f(x) = 5$ and $f(2) = 3$. The f^n $f(x)$ only needs to be arbitrarily close to 5, not defined for 5.

Ex:

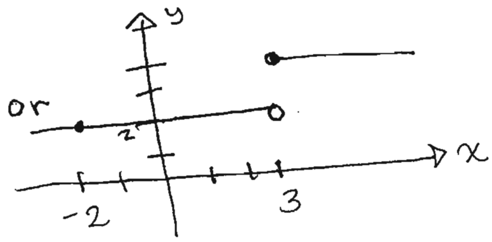


$$f(x) = \begin{cases} g(x) & \forall x \neq 2 \\ 3 & x = 2 \end{cases}$$

2.



or



3. a) $\lim_{x \to -3} \dfrac{x^2 - x + 12}{x+3}$

D.S.: $\dfrac{9+3+12}{0} = \dfrac{24}{0}$ undef.

→ check one sided limits

$\lim_{x \to -3^-} \dfrac{x^2 - x + 12}{x+3}$ $\dfrac{(+)+(+)+(+)}{(-)} \to -\infty$

$\lim_{x \to -3^+} \dfrac{x^2 - x + 12}{x+3}$ $\dfrac{(+)+(+)+(+)}{(+)} \to \infty$

$\boxed{\text{the limit DNE}}$

b) $\lim_{t \to 0} \dfrac{\sqrt{2-t} - \sqrt{2}}{t}$

D.S.: $\dfrac{\sqrt{2} - \sqrt{2}}{0} = \dfrac{0}{0}$

use replacement Th^m.

$$\frac{\sqrt{2-t} - \sqrt{2}}{t} \cdot \frac{\sqrt{2-t} + \sqrt{2}}{\sqrt{2-t} + \sqrt{2}} = \frac{2 - t - 2}{t(\sqrt{2-t} + \sqrt{2})} = \frac{-\cancel{t}}{\cancel{t}(\sqrt{2-t} + \sqrt{2})}$$

$$\to \lim_{t \to 0} \frac{-1}{\sqrt{2-t} + \sqrt{2}} = \frac{-1}{\sqrt{2} + \sqrt{2}} = \boxed{-\frac{1}{2\sqrt{2}}}$$

c) $\lim_{h \to 0} \dfrac{(h-5)^2 - 25}{h}$

D.S.: $\dfrac{0}{0}$

use replacement Th^m

$$\frac{(h-5)^2 - 25}{h} = \frac{h^2 - 10h + 25 - 25}{h} = \frac{h(h-10)}{h} = h - 10$$

$$\to \lim_{h \to 0} h - 10 = \boxed{-10}$$

d) ~~Same as~~ is problem a

3. e) $\lim\limits_{x \to -4} |x+4|$

use def. of absolute value f^n

$$|x+4| = \begin{cases} (x+4) & x \geq -4 \\ -(x+4) & x < -4 \end{cases}$$

$$\left.\begin{aligned} \lim_{x \to -4^-} -(x+4) &= 0 \\ \lim_{x \to -4^+} (x+4) &= 0 \end{aligned}\right\} \lim_{x \to -4} |x+4| = 0$$

f) $\lim\limits_{x \to 1} \dfrac{x^2-1}{|x-1|}$

$$|x-1| = \begin{cases} x-1 & x \geq 1 \\ -(x-1) & x < 1 \end{cases}$$

$$\left.\begin{aligned} \lim_{x \to 1^-} \frac{(x-1)(x+1)}{-(x-1)} &= \lim_{x \to 1^-} \frac{x+1}{-1} = -2 \\ \lim_{x \to 1^+} \frac{(x-1)(x+1)}{(x-1)} &= \lim_{x \to 1^+} x+1 = 2 \end{aligned}\right\} \text{limit DNE}$$

g) $\lim\limits_{x \to 1} \left[\dfrac{1}{x-1} - \dfrac{2}{x^2-1}\right]$ D.S.: $\frac{1}{0} - \frac{2}{0} = \infty - \infty$ indeterminate form

$$\to \lim_{x \to 1} \left[\frac{x+1-2}{x^2-1}\right] = \lim_{x \to 1} \frac{x-1}{(x-1)(x+1)} = \lim_{x \to 1} \frac{1}{x+1} = \boxed{\frac{1}{2}}$$

4) (a) $\lim\limits_{x \to 0^+} x^2 = 0$

(b) $\lim\limits_{x \to 0^-} x = 0$, w/ (a) $\Rightarrow \lim\limits_{x \to 0} h(x) = 0$

(c) $\lim\limits_{x \to 2^-} x^2 = 4$

(d) $\lim\limits_{x \to 1} x^2 = 1$

(e) $\lim\limits_{x \to 2^+} 8 - x = 6$

(f) Limit DNE from results of parts c & e

5) a) $\lim\limits_{x \to 3} \dfrac{1}{(x-3)^8} \to$ D.S. $\to \frac{1}{0}$

look at one-sided limits

$$\left.\begin{aligned} \lim_{x \to 3^-} \frac{1}{(x-3)^8} \quad \frac{(+)}{(+)} &\to \infty \\ \lim_{x \to 3^+} \frac{1}{(x-3)^8} \quad \frac{(+)}{(+)} &\to \infty \end{aligned}\right\} \lim_{x \to 3} \frac{1}{(x-3)^8} = \infty$$

Handout 3

5b) $\lim_{x \to 0} \frac{x-1}{x^2(x+2)}$ D.S. $\to \frac{-1}{0}$ undefined

look at one-sided limits

$\lim_{x \to 0^-} \frac{x-1}{x^2(x+2)}$ $\frac{(-)}{(+)(+)}$ $\frac{(-)}{(+)} \to -\infty$

$\lim_{x \to 0^+} \frac{x-1}{x^2(x+2)}$ $\frac{(-)}{(+)(+)} \to -\infty$

$\lim_{x \to 0} \frac{x-1}{x^2(x+2)} = -\infty$

6) a) we know

$-1 \le \cos(20\pi x) \le 1$

$-x^2 \le x^2 \cos(20\pi x) \le x^2$

$\lim_{x \to 0} -x^2 = 0$

$\lim_{x \to 0} x^2 = 0$

$\Rightarrow$ by Squeeze Th^m, $\lim_{x \to 0} x^2 \cos(20\pi x) = 0$

b) we know

$-1 \le \sin(\pi/x) \le 1$

$e^{-1} \le e^{\sin(\pi/x)} \le e^{1}$

$\sqrt{x}\, e^{-1} \le \sqrt{x}\, e^{\sin(\pi/x)} \le \sqrt{x} \cdot e$

$\lim_{x \to 0^+} \sqrt{x} \cdot e^{-1} = 0$

$\lim_{x \to 0^+} \sqrt{x}\, e^{1} = 0$

by Squeeze Th^m,

$\lim_{x \to 0^+} \sqrt{x} \cdot e^{\sin(\pi/x)} = 0$