Handout 4 - 2.5, 2.6, 2.7

1) $$\lim_{x \to -\infty} \left(5 + \frac{100}{x} + \frac{\sin^4(x^3)}{x^2}\right) = 5 + \frac{100}{-\infty} + \frac{\sin^4(-\infty)}{(-\infty)^2}$$

$$= 5 + 0 + \frac{[-1,1]}{\infty} = 5 + 0 = 5$$

2) $$\lim_{x \to \infty} \frac{4x^2 - 7}{8x^2 + 5x + 2} = \frac{\infty}{\infty}$$

Use x^2: $$\lim_{x \to \infty} \frac{\frac{4x^2}{x^2} - \frac{7}{x^2}}{\frac{8x^2}{x^2} + \frac{5x}{x^2} + \frac{2}{x^2}} = \lim_{x \to \infty} \frac{4 - \frac{7}{x^2}}{8 + \frac{5}{x} + \frac{2}{x^2}} = \frac{4 - 0}{8 + 0 + 0} = \frac{1}{2}$$

3) $$\lim_{x \to \infty} \frac{\sqrt{36x^2 + 2x + 7}}{3x}$$

use: x

$$\lim_{x \to \infty} \frac{\frac{1}{x}}{\frac{1}{x}} \cdot \frac{\sqrt{36x^2 + 2x + 7}}{3x}$$

$$= \lim_{x \to \infty} \frac{\frac{1}{\sqrt{x^2}} \cdot \frac{\sqrt{36x^2 + 2x + 7}}{1}}{\frac{1}{x} \cdot \frac{3x}{1}} = \lim_{x \to \infty} \frac{\sqrt{\frac{36x^2}{x^2} + \frac{2x}{x^2} + \frac{7}{x^2}}}{\frac{3x}{x}} = \lim_{x \to \infty} \frac{\sqrt{36 + \frac{2}{x} + \frac{7}{x^2}}}{3}$$

$$= \frac{\sqrt{36 + 0 + 0}}{3} = \frac{6}{3} = 2$$

4) We want $\lim_{x \to 3^-} f(x) = \lim_{x \to 3^+} f(x)$ so $\lim_{x \to 3} f(x)$ exists.

or we want ~~all~~ the two piecewise functions to have the same value at $x = 3$.

Either gives us continuity because the ~~functions~~ functions are polynomials.

$$\lim_{x \to 3^-} f(x) = \lim_{x \to 3^+} f(x)$$

$$\lim_{x \to 3^-} x^3 + k = \lim_{x \to 3^+} kx - 5$$

use direct substitution

$$(3)^3 + k = k(3) - 5$$

$$2k = -32$$

$$\boxed{k = -16}$$

5) $f(x)$ is discontinuous at $\left.\begin{matrix} x=-1/2 \\ x=-2 \end{matrix}\right\}$ b/c $\begin{matrix} 2x+1 \neq 0 \\ 3x+6 \neq 0 \end{matrix}$

for $x=-1/2$ $\quad \lim_{x \to -\frac{1}{2}} f(x) = \frac{1}{0}$ by D.S.

look at one-sided limits

$$\left.\begin{matrix} \lim_{x \to -\frac{1}{2}^-} \dfrac{-2x}{(2x+1)(3x+6)} \quad \dfrac{(+)}{(-)(+)} \to -\infty \\ \lim_{x \to -\frac{1}{2}^+} \dfrac{-2x}{(2x+1)(3x+6)} \quad \dfrac{(+)}{(+)(+)} \to \infty \end{matrix}\right\} \text{the limit DNE}$$

for $x=-2$ $\quad \lim_{x \to -2} f(x) = \frac{1}{0}$ by D.S.

look at one-sided limits

$$\left.\begin{matrix} \lim_{x \to -2^-} \dfrac{-2x}{(2x+1)(3x+6)} \quad \dfrac{(+)}{(-)(-)} \to \infty \\ \lim_{x \to -2^+} \dfrac{-2x}{(2x+1)(3x+6)} \quad \dfrac{(+)}{(-)(+)} \to -\infty \end{matrix}\right\} \text{the limit DNE}$$

Use the limit proof to show each limit exists

6) $\lim_{x \to 2} 3x+5 = 11$

$L=11$, $a=2$

$|3x+5-11| < \varepsilon$

$|3x-6| < \varepsilon$

$3|x-2| < \varepsilon$

$|x-2| < \varepsilon/3$

If $\delta = \varepsilon/3$, then $|f(x)-L| < \varepsilon$

and $\lim_{x \to 2} f(x) = 11$ is true

7) $\lim_{x \to 1} \dfrac{5x+3}{4} = 2$

$L=2$, $a=1$

$\left|\dfrac{5x+3}{4} - 2\right| < \varepsilon$

$\left|\dfrac{5x+3-8}{4}\right| < \varepsilon$

$\left|\dfrac{5x-5}{4}\right| < \varepsilon$

$\left|\frac{5}{4}(x-1)\right| < \varepsilon$

$|(x-1)| < \dfrac{4\varepsilon}{5}$

If $\delta = \dfrac{4\varepsilon}{5}$, then $|f(x)-L| < \varepsilon$

and $\lim_{x \to 1} f(x) = 2$ is true