Below are solutions to selected problems from the homework assignment. Note, I can add to these solutions if other problems are requested.

1 Homework 1 - Assigned Problems

1.1 Section 1.3

1.1.1 1.3.41

Solve without using a calculator.

$$10^{x^2-4} = 1$$

Solution.

We have two options:

1. Make it so both sides of the equation has the same exponential base.

$$\begin{aligned} 10^{x^2-4} &= 1 \\ 10^{x^2-4} &= 10^0 \\ x^2 - 4 &= 0 \\ x &= \pm 2 \end{aligned}$$

2. Use logarithms to get rid of the exponential function.

$$\begin{aligned} 10^{x^2-4} &= 1 \\ \log_{10}\left(10^{x^2-4}\right) &= \log_{10}(1) \\ x^2 - 4 &= 0 \qquad \text{since } \log_b(1) = 0 \text{ for any base } b \\ x &= \pm 2 \end{aligned}$$

Note that the base of the logarithm and the exponential function must be the same in order for the inverse property to hold. Also, we use the same log on both sides of the equation.

1.1.2 1.3.47

Solve without using a calculator.

$$3^{3x-4} = 15$$

Solution.

Unfortunately, we cannot make both sides have the same base. We can use logarithms to get rid of the exponential function (since they are inverses of each other).

$$\begin{aligned} 3^{3x-4} &= 15 \\ \log_3\left(3^{3x-4}\right) &= \log_3(15) && \text{apply the log to both sides} \\ 3x-4 &= \log_3(15) && \text{reduce the left side using inverse prop.} \\ 3x &= 4+\log_3(15) \\ x &= \frac{4+\log_3(15)}{3} && \text{no other simplication is necessary} \end{aligned}$$

2 Homework 1 - Suggested Problems

2.1 Section 1.3

2.1.1 1.3.44

Solve without using a calculator.

$$9^x + 3^{x+1} - 18 = 0$$

Solution.

We will:

1. Write the terms using 3^x
2. Use the substitution $u = 3^x$

$$\begin{aligned} 9^x + 3^{x+1} - 18 &= 0 \\ 3^{2x} + 3\cdot 3^x - 18 &= 0 \\ u^2 + 3u - 18 &= 0 && \text{use the substitution} \\ (u+6)(u-3) &= 0 && \text{factor} \\ u = -6 \qquad & u = 3 \\ 3^x = -6 \qquad & 3^x = 3 && \text{replace the substitution} \\ 3^x \neq -6 \qquad & x = 1 && \text{remember, exponentials are non-negative} \end{aligned}$$

So the solution is $x = 1$.

2.1.2 1.3.48

Solve without using a calculator.

$$5^{3x} = 29$$

Solution.

This problem is similar to 1.3.47. We can't make both sides of the equation have the same exponential base, so we'll use logarithms.

$$\begin{aligned} 5^{3x} &= 29 && \\ \log_5\left(5^{3x}\right) &= \log_5(29) && \text{Take the log of both sides} \\ 3x &= \log_5(29) && \text{use inverse prop.} \\ x &= \frac{\log_5(29)}{3} && \end{aligned}$$