Below are solutions to selected problems from the homework assignment. Note, I can add to these solutions if other problems are requested.

1 Homework 2 - Assigned Problems

Section 1.4

Problem 1.4.33

Solve the following

$$\cos(3x) = \sin(3x) \qquad 0 \leq x < 2\pi$$

Solution.

We can use the substitution $u = 3x$ so we don't have to work with a function that is horizontally scaled. However, this changes the interval for our solution:

$$0 \leq x < 2\pi \quad \to \quad 0 \leq \frac{u}{3} < 2\pi \quad \to \quad 0 \leq u < 6\pi$$

So, we can solve for u

$$\begin{aligned} \cos(u) &= \sin(u) \qquad \text{True only for } u = \frac{\pi}{4}, \frac{5\pi}{4} \\ \text{or divide both sides by } \cos(u) & \\ 1 &= \tan(u) \qquad \text{True for } u = \frac{\pi}{4}, \frac{5\pi}{4} \end{aligned}$$

We want to find all solutions within $0 \leq u < 6\pi$. So, we'll add 2π to each answer. Note, we'll use $2\pi = \frac{8\pi}{4}$ and will keep solutions that are less than $6\pi = \frac{24\pi}{4}$

$$u = \begin{cases} & \frac{\pi}{4} \\ \frac{\pi}{4} + \frac{8\pi}{4} = & \frac{9\pi}{4} \\ \frac{\pi}{4} + 2 \cdot \frac{8\pi}{4} = & \frac{17\pi}{4} \end{cases} \qquad u = \begin{cases} & \frac{5\pi}{4} \\ \frac{5\pi}{4} + \frac{8\pi}{4} = & \frac{13\pi}{4} \\ \frac{5\pi}{4} + 2 \cdot \frac{8\pi}{4} = & \frac{21\pi}{4} \end{cases}$$

So the solutions are:

$$u = \frac{\pi}{4}, \frac{9\pi}{4}, \frac{17\pi}{4}, \frac{5\pi}{4}, \frac{13\pi}{4}, \frac{21\pi}{4}$$

Since $u = 3x$, we have to divide all of our answers by 3:

$$x = \frac{\pi}{12}, \frac{9\pi}{12}, \frac{17\pi}{12}, \frac{5\pi}{12}, \frac{13\pi}{12}, \frac{21\pi}{12}$$

Section 2.1

Problem 2.1.15

Make a table of average velocities and make a conjecture about the instantaneous velocity at the indicated time.

$$s(t) = 40\sin(2t) \quad \text{at } t = 0$$

Solution.

Average velocity between points $(a, s(a))$ and $(b, s(b))$ is given by

$$\frac{s(b) - s(a)}{b - a}$$

Time Interval	$[0, 1]$	$[0, 0.5]$	$[0, 0.1]$	$[0, 0.01]$	$[0, .001]$
Average Velocity	36.3719	67.3177	79.4677	79.9947	79.999

Recall that instantaneous velocity is the limit of average velocity. From the table, we see that the instantaneous velocity is 80.

Problem 2.1.17

Make a table of slopes of secant lines and make a conjecture about the slope of the tangent line at the indicated point.

$$f(x) = 2x^2 \quad \text{at } x = 2$$

Solution.

Slope of a secant line between points $(a, f(a))$ and $(b, f(b))$ is given by

$$\frac{f(b) - f(a)}{b - a}$$

Time Interval	$[1, 2]$	$[1.5, 2]$	$[1.9, 2]$	$[1.99, 2]$	$[1.999, 2]$
Secant Line Slopes	6	7	7.8	7.98	7.998

Note that the slope of a tangent line is the limit of the slopes of secant lines. From the table, the slope of the tangent line is 8.

Section 2.2

Problem 2.2.21

a. Make a table of values of $\sin\left(\frac{1}{x}\right)$ for $x = \frac{2}{\pi}, \frac{2}{3\pi}, \frac{2}{5\pi}, \frac{2}{7\pi}, \frac{2}{9\pi}, \frac{2}{11\pi}$. Describe the pattern of values you observe.

Solution.

x	$2/\pi$	$2/3\pi$	$2/5\pi$	$2/7\pi$	$2/9\pi$	$2/11\pi$
$\sin(1/x)$	1	-1	1	-1	1	-1

The values alternate from 1 and -1.

b. Why does a graphing utility have difficulty plotting the graph of $y = \sin\left(\frac{1}{x}\right)$ near $x = 0$?

Solution.

The function $y = \sin\left(\frac{1}{x}\right)$ near $x = 0$ oscillates between all the values in the interval $[-1, 1]$ an infinite number of times.

c. What do you conclude about $\lim_{x\to 0} \sin(1/x)$?

Solution.

The limit does not exist because the function oscillates between the values in the interval $[-1, 1]$ instead of approaching a single value.