Below are solutions to selected problems from the homework assignment. Note, I can add to these solutions if other problems are requested.

1 Homework 5 - Assigned Problems

Section 3.3

Problem 3.3.25

a Use the quotient rule to find the derivative of the given function. Simplify your result.

b Find the derivative by first simplifying the function. Verify that your answer agrees with part(a).

Solution.

Part (a): Using the quotient rule, we have to find the derivatives of the top and bottom functions *separately*.

$$f(x) = \frac{x-a}{\sqrt{x}-\sqrt{a}} = \frac{h(x)}{g(x)} \qquad a\text{is a positive constant}$$

$$\begin{aligned} h(x) &= x - a & g(x) &= \sqrt{x} - \sqrt{a} \\ h'(x) &= 1 - 0 & g'(x) &= \frac{1}{2}x^{-1/2} - 0 \end{aligned}$$

By the quotient rule, the derivative is

$$\begin{aligned} f'(x) &= \frac{h'(x)\cdot g(x) - h(x)\cdot g'(x)}{[g(x)]^2} \\ &= \frac{1\cdot(\sqrt{x}-\sqrt{a}) - (x-a)\cdot\left(\frac{1}{2}x^{-1/2}\right)}{[\sqrt{x}-\sqrt{a}]^2} \\ &= \frac{(\sqrt{x}-\sqrt{a})\left(1-(\sqrt{x}+\sqrt{a})\,\frac{1}{2}x^{-1/2}\right)}{[\sqrt{x}-\sqrt{a}]^2} \qquad x-a = \left(\sqrt{x}-\sqrt{a}\right)\left(\sqrt{x}+\sqrt{a}\right) \\ &= \frac{\left(1-(\sqrt{x}+\sqrt{a})\,\frac{1}{2}x^{-1/2}\right)}{\sqrt{x}-\sqrt{a}} \\ &= \frac{\left(1-\frac{1}{2}-\frac{1}{2}\frac{\sqrt{a}}{\sqrt{x}}\right)}{\sqrt{x}-\sqrt{a}} \\ &= \frac{\left(\frac{1}{2}-\frac{1}{2}\frac{\sqrt{a}}{\sqrt{x}}\right)}{\sqrt{x}-\sqrt{a}} = \frac{\left(\frac{1}{2}-\frac{1}{2}\frac{\sqrt{a}}{\sqrt{x}}\right)}{\sqrt{x}-\sqrt{a}} = \frac{\frac{1}{2\sqrt{x}}\left(\sqrt{x}-\sqrt{a}\right)}{\sqrt{x}-\sqrt{a}} = \frac{1}{2\sqrt{x}} \end{aligned}$$

Solution.

Part (b): We can simplify using two methods.

$$f(x) = \frac{x-a}{\sqrt{x}-\sqrt{a}} \cdot \frac{\sqrt{x}+\sqrt{a}}{\sqrt{x}+\sqrt{a}} = \frac{(x-a)\left(\sqrt{x}+\sqrt{a}\right)}{(x-a)} = \sqrt{x}+\sqrt{a}$$

Or

$$f(x) = \frac{x-a}{\sqrt{x}-\sqrt{a}} = \frac{(\sqrt{x}+\sqrt{a})(\sqrt{x}+\sqrt{a})}{\sqrt{x}-\sqrt{a}} = \sqrt{x}+\sqrt{a}$$

We can take the derivative now.

$$f'(x) = \frac{1}{2}x^{-1/2} + 0 = \frac{1}{2\sqrt{x}}$$

Problem 3.3.35

Find the derivative of the following function.

$$g(t) = \frac{t^3+3t^2+t}{t^3}$$

Solution.

We can split the function up into three fractions. Then, we can simplify the fractions.

$$\begin{aligned} g(t) &= \frac{t^3+3t^2+t}{t^3} \\ &= \frac{t^3}{t^3} + \frac{3t^2}{t^3} + \frac{t}{t^3} \\ &= 1 + \frac{3}{t} + \frac{1}{t^2} \\ &= 1 + 3t^{-1} + t^{-2} \end{aligned}$$

Note that we *must* rewrite the terms so that they are not in the denominators. This way, we can use the power rule for finding derivatives.

$$\begin{aligned} g'(t) &= 0 + (-1)\cdot 3t^{-2} + (-2)t^{-3} \\ &= -3t^{-2} - 2t^{-3} \end{aligned}$$

Problem 3.3.49

Compute the derivative of the following function.

$$g(x) = \frac{x(3-x)}{2x^2}$$

Solution.

Let the function in the numerator be $f(x) = x(3-x)$ and function in the denominator be $h(x) = 2x^2$.

$$\begin{aligned} f(x) &= x(3-x) \\ f'(x) &= 1 \cdot (3-x) + x(0-1) \\ &= 3 - x - x \\ &= 3 - 2x \end{aligned} \qquad \begin{aligned} h(x) &= 2x^2 \\ h'(x) &= 4x \end{aligned}$$

Using the quotient rule, we have

$$\begin{aligned} g'(x) &= \frac{f'(x) \cdot h(x) - f(x) \cdot h'(x)}{[h(x)]^2} \\ &= \frac{(3-2x)(2x^2) - x(3-x)(4x)}{[2x^2]^2} \\ &= \frac{6x^2 - 4x^3 - 12x^2 + 4x^3}{4x^4} \\ &= \frac{-6x^2}{4x^4} \\ &= \frac{-3}{2x^2} \end{aligned}$$