1 Homework 7 - Assigned Problems

Section 3.7

3.7.17

Use implicit differentiation to find $\frac{dy}{dx}$.

$$6x^3 + 7y^3 = 13xy$$

Solution.

1. We have to use "Chain Rule" for the $7y^3$ term
2. We have to use product rule for the $13xy$ term

First we differentiate the equation.

$$\begin{aligned} 6 \cdot 3 \cdot x^2 + 7 \cdot 3 \cdot y^2 \cdot \frac{dy}{dx} &= 13 \cdot y + 13x \cdot \frac{dy}{dx} \\ 18 \cdot x^2 + 21 \cdot y^2 \cdot \frac{dy}{dx} &= 13 \cdot y + 13x \cdot \frac{dy}{dx} \end{aligned}$$

Now we collect all the $\frac{dy}{dx}$ terms on one side of the equation and all the non-$\frac{dy}{dx}$ terms on the *other* side.

$$\begin{aligned} 18 \cdot x^2 + {\color{red}21 \cdot y^2 \cdot \frac{dy}{dx}} &= 13 \cdot y + {\color{red}13x \cdot \frac{dy}{dx}} \\ 18 \cdot x^2 - 13 \cdot y &= 13x \cdot \frac{dy}{dx} - 21 \cdot y^2 \cdot \frac{dy}{dx} \\ 18 \cdot x^2 - 13 \cdot y &= \frac{dy}{dx}\left[13x - 21 \cdot y^2\right] \qquad \text{Factor } \frac{dy}{dx} \\ \frac{18x^2 - 13y}{13x - 21y^2} &= \frac{dy}{dx} \\ \frac{13y - 18x^2}{21y^2 - 13x} &= \frac{dy}{dx} \qquad \text{Factor -1 from top and bottom} \end{aligned}$$

3.7.37

Find $\frac{dy}{dx}$ of

$$y = \sqrt[4]{\frac{2x}{4x-3}}$$

Solution.

To match the answer in the back of the book, we have to do a lot of clever algebra.

$$\begin{aligned}
y &= \left(\frac{2x}{4x-3}\right)^{1/4} \\
y' &= \frac{1}{4}\left(\frac{2x}{4x-3}\right)^{-3/4} \cdot \left(\frac{2\cdot(4x-3)-(2x)(4)}{(4x-3)^2}\right) && \text{chain and quotient rules} \\
&= \frac{1}{4}\left(\frac{2x}{4x-3}\right)^{-3/4} \cdot \left(\frac{8x-6-8x}{(4x-3)^2}\right) \\
&= \frac{1}{4}\left(\frac{2x}{4x-3}\right)^{-3/4} \cdot \left(\frac{-6}{(4x-3)^2}\right) \\
&= \frac{1}{4}\frac{(2x)^{-3/4}}{(4x-3)^{-3/4}} \cdot \frac{-6}{(4x-3)^2} \\
&= \frac{(2x)^{-3/4}}{4} \cdot \frac{-6}{(4x-3)^{2-3/4}} && \text{Combine the binomial terms of } (4x-3) \\
&= \frac{(2^{-3/4}x^{-3/4})}{2} \cdot \frac{-3}{(4x-3)^{5/4}} \\
&= \frac{1}{2^{7/4}x^{3/4}} \cdot \frac{-3}{(4x-3)^{5/4}}
\end{aligned}$$

Section 3.8

Problem 3.8.15

Find the derivative. Give the interval on which the results are valid.

$$\frac{d}{dx}\left[\ln\left(\ln x\right)\right]$$

Solution.

1. The domain of the function above is $x > 1$.

   The domain of the inner $\ln x$ is $x > 0$. However, the inner $\ln x$ will be negative for $x < 1$. The positive values of $\ln x$ occur when $x > 1$. Thus the domain is when $x > 1$.

2. We have to use the chain rule to find the derivative.

$$\begin{aligned}\frac{d}{dx}\Big[\ln(\ln x)\Big] &= \frac{1}{\ln x} \cdot \frac{d}{dx}\Big[\ln(x)\Big] \\ &= \frac{1}{\ln x} \cdot \frac{1}{x}\end{aligned}$$

Problem 3.8.27

Use the General Power rule to find the derivative of the function.

$$g(y) = e^y \cdot y^e$$

Solution.

Note that this function is dependent on variable y. So, we'll find the derivative *with respect to* y.

$$\begin{aligned}\frac{d}{dy}\big[g(y)\big] &= \frac{d}{dy}\big[e^y \cdot y^e\big] \\ g'(y) &= e^y \cdot y^e + e^y \cdot e \cdot y^{e-1} && \text{By Product Rule} \\ g'(y) &= e^y \cdot y^e + e^{y+1} \cdot y^{e-1} && \text{Combine similar bases}\end{aligned}$$

Problem 3.8.45

Find the derivative of $f(x) = x^{\ln x}$

Solution.

We have to use $\ln(x)$ on both sides of the equation we are given to find the derivative. The power rule does not work here because we are not working with a power function of the form x^n, $n \neq 0$ is any real number.

$$\begin{aligned}
y &= x^{\ln(x)} \\
\ln(y) &= \ln\left(x^{\ln(x)}\right) && \text{Apply } \ln(x) \text{ on both sides} \\
\ln(y) &= \ln(x) \cdot \ln(x) && \text{Use power property of logs.} \\
\frac{1}{y} \cdot \frac{dy}{dx} &= \ln(x) \cdot \frac{1}{x} + \ln(x) \cdot \frac{1}{x} && \text{Using product rule.} \\
\frac{dy}{dx} &= y \cdot 2\frac{\ln(x)}{x} && \text{Multiply both sides by } y \\
\frac{dy}{dx} &= x^{\ln(x)} \cdot 2\frac{\ln(x)}{x} && \text{Replace with original equation}
\end{aligned}$$