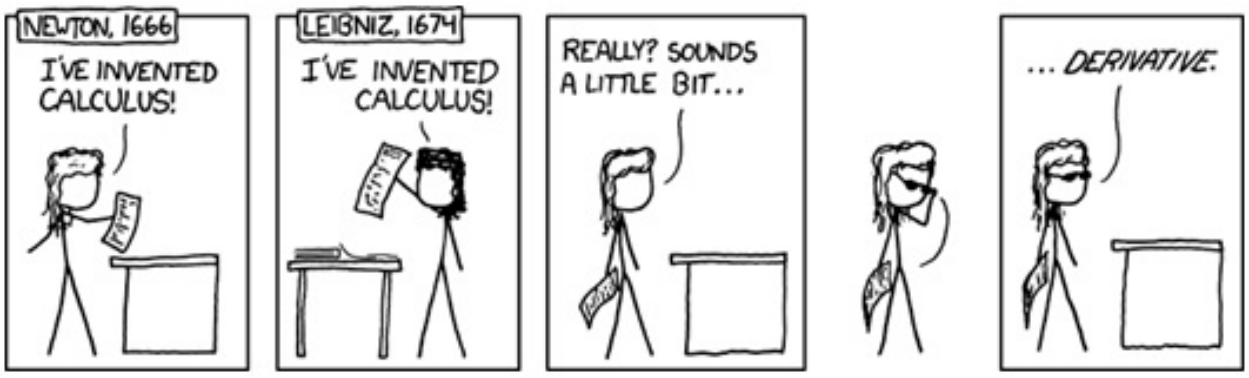
Exam 1

Name: _____

Signature: _____

Student ID: _____

- No notes or books.
- Show all work.
- Good luck!



1. (6 points) Find the center and radius of the sphere given below.

$$x^2 + y^2 + z^2 - 6x + 4y - 8z - 2 = 0$$

2. (9 points) Consider the vectors $\mathbf{u} = \langle 5, -1, 2 \rangle$ and $\mathbf{v} = \langle 1, -3, 0 \rangle$.

   (a) (5 points) Calculate $\text{proj}_{\mathbf{v}} \mathbf{u}$.

   (b) (4 points) Find the unit vector of $\mathbf{u}$.

3. (15 points) The vector-valued equation of velocity for a strange object is given as

$$\mathbf{v}(t) = \left\langle \frac{4}{t-1}, e^{t-2}, \pi \sin(\pi t) \right\rangle \qquad t > 1$$

(a) (6 points) Find the object's vector-valued equation of acceleration.

(b) (9 points) Find the object's vector-valued equation of motion if we know the object passes through $\mathbf{r}(2) = \langle -5, 3, -2 \rangle$

4. (7 points) Find the arc length function $s(t)$ for $\mathbf{r}(t) = \langle 3t + 1, 4t - 5, 2t \rangle$.

5. (9 points) Find the curvature of $\mathbf{r}(t) = \left\langle e^{-2t}, \frac{1}{2}t^2, 4 \right\rangle$ at $t = 0$. (Hint: use the cross product version of the curvature.)

6. (8 points) An airplane flies northwest at a constant altitude at 550 mi/hr relative to the air. There is a southernly crosswind with a magnitude of 40 mi/hr. Find the velocity vector of the airplane relative to the ground. You may leave your answer in terms of radicals.

7. (6 points) Let $\mathbf{u} = \langle u_1, u_2 \rangle$ and $\mathbf{v} = \langle v_1, v_2 \rangle$. Show that $(\mathbf{u} + \mathbf{v}) \bullet (\mathbf{u} + \mathbf{v}) = |\mathbf{u}|^2 + |\mathbf{v}|^2$ if $\mathbf{u}$ and $\mathbf{v}$ are perpendicular. (Hint: start with the left side of the equation and rewrite it.)

   NOTE: This is a proof. Do NOT use a vector with specific values.

EXTRA. (4 points) Let a and b be scalars and $\mathbf{u} = \langle u_1, u_2, u_3 \rangle$ and $\mathbf{v} = \langle v_1, v_2, v_3 \rangle$ be vectors. Prove the following: $(a\mathbf{u}) \times (b\mathbf{v}) = (ab)(\mathbf{u} \times \mathbf{v})$. (NOTE: This is a proof. Do not use vectors with specific values.)

Problem	Max Points	Points
1	6	
2	9	
3	15	
4	7	
5	9	
6	8	
7	6	
Extra	4	
Total	60	