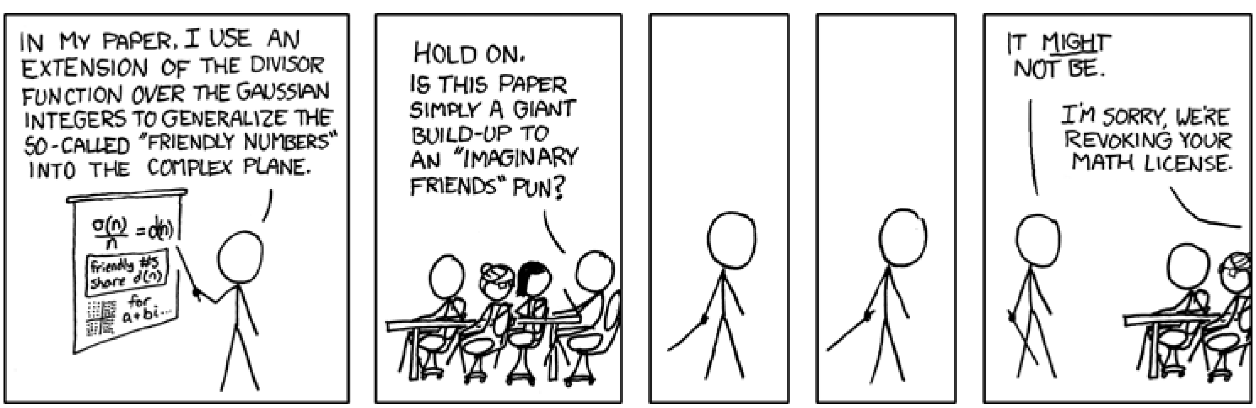
Exam 2

Name: ______________________________

Signature: ______________________________

Student ID: ______________________________

- No notes or books.
- Show all work.
- Good luck!



1. (12 points) Consider the surface below.

$$x^2 + \frac{y^2}{4} + z^2 = 1$$

   (a) (6 points) Find the xy, xz, and yz traces of the surface.

   (b) (1 point) What type of surface is the function?

   (c) (5 points) Find the equation of the tangent plane to the surface at the point $(0, 2, 0)$.

2. (9 points) Find the partial derivatives f_x, f_y, f_z for the function below.
   DO NOT SIMPLIFY YOUR ANSWER.

$$f(x, y, z) = \frac{x^2 + y}{xyz + z^2}$$

3. (5 points) For the following set of three planes, determine if the planes are parallel, orthogonal, or none of the above.

$$Q: \ -2x - 2y + 4z = 2 \qquad R: \ x + y + z = 1 \qquad S: \ -2x - 2y - 2z = 15$$

4. (12 points) Consider the function below.

$$f(x, y) = \frac{2y}{x^2 + y}$$

(a) (2 points) Compute the limit as $(x, y) \to (0, 0)$ along the y-axis.

(b) (2 points) Compute the limit as $(x, y) \to (0, 0)$ along the line $y = mx$, $m \neq 0$

(c) (2 points) Compute the limit as $(x, y) \to (0, 0)$ along the parabola $y = x^2$.

(d) (3 point) What can you conclude about the limit at $(0, 0)$? Explain why.

(e) (3 points) What can you conclude about the continuity of the function below at $(0, 0)$? Explain your reasoning.

$$g(x, y) = \begin{cases} \frac{2y}{x^2+y} & \text{if } (x, y) \neq (0, 0) \\ 3 & \text{if } (x, y) = (0, 0) \end{cases}$$

5. (12 points) The area of a right triangle with base b and height h is $A = \frac{1}{2}bh$.

   (a) (4 points) Assume that b and h change according to the function $b = g(t)$ and $h = f(t)$. Find $\frac{dA}{dt}$.

   (b) (6 points) Suppose that the sides of the triangle oscillate in length according to $b = 1 + \cos^2(t)$ and $h = 1 + \sin^2(t)$. Find $\frac{dA}{dt}$.

   (c) (2 points) Is the area of the triangle in part (b) increasing or decreasing at $t = \frac{\pi}{2}$?

6. (12 points) Use the Second Derivative Test to classify the critical point(s) of the function below.

   Note: You do NOT have to compute function values.

$$f(x, y) = x^2 + y^2 + x^2y + 4$$

7. (8 points) Using Lagrange multipliers, find the point at which the maximum occurs for the given function and constraint.

$$f(x, y) = 12xy - x^2 - 3y^2 \qquad \text{subject to} \quad x + y = 16$$

EXTRA. (4 points) Find the limit (it does exist).

$$\lim_{(x,y)\to(4,0)} x^2 y \ln(xy)$$

Problem	Max Points	Points
1	12	
2	9	
3	5	
4	12	
5	12	
6	12	
7	8	
Extra	4	
Total	70	