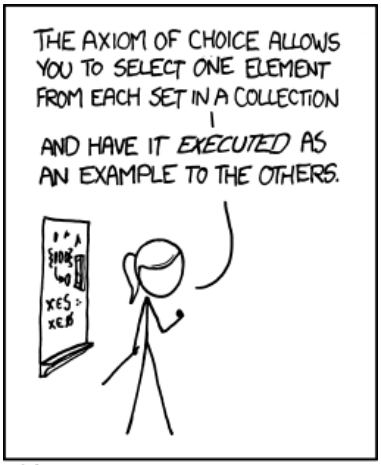
Exam 3

Name: _____

Signature: _____

Student ID: _____

- No notes or books.
- Show all work.
- Good luck!



1. (8 points) Consider the double integral. $\int_0^4 \int_{\sqrt{x}}^2 \frac{1}{y^3+1}\, dy\, dx$

   (a) (2 points) Why is the current order of integration not desirable?

   (b) (6 points) Change the order of integration. DO NOT EVALUATE.

2. (6 points) Change the order of integration for the triple integral using the given order. DO NOT EVALUATE.

$$\int_0^3 \int_0^4 \int_0^{\sqrt{y}} f(x,y,z)\, dz\, dy\, dx \quad \text{use order } dy\, dx\, dz$$

3. (10 points) Evaluate the given integral by first changing to polar coordinates. (**NO** trigonometric substitutions needed.)

$$\int\int_R x+y\, dA \qquad R \text{ is the region such that } y \geq 0 \text{ and is between the circles } \begin{array}{l} x^2+y^2=1 \\ x^2+y^2=4 \end{array}$$

4. (12 points) A thin plate bounded by $x = 0, x = 4, y = 0,$ and $y = 2$ has a density of $\rho(x, y) = 1 + x$. Find its center of mass.

5. (9 points) Write the integral below using cylindrical coordinates. Note that the solid we are integrating over is a cylinder with height 2. DO NOT EVALUATE.

$$\int_0^2 \int_{-1}^{1} \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} (x^2 + y^2 + z^2)\, dy\, dx\, dz$$

6. (9 points) Write the integral below using spherical coordinates. Note that the solid we are integrating over is a sphere of radius 1. DO NOT EVALUATE.

$$\int_{-1}^{1} \int_{-\sqrt{1-z^2}}^{\sqrt{1-z^2}} \int_{-\sqrt{1-x^2-z^2}}^{\sqrt{1-x^2-z^2}} \left(x^2 + y^2 + z^2\right)^{3/2}\, dy\, dx\, dz$$

7. (8 points) Find the average value of $f(x, y)$ over the region R.

$f(x, y) = xy$ R is the triangle with vertices $(0, 0), (1, 0),$ and $(1, 3)$

8. (8 points) Make the appropriate change of variables for the integral below. DO NOT EVALUATE.

$\int\int_R (x+y)e^{x^2-y^2}\, dA$ R is the rectangle enclosed by the lines $x - y = 0$ $x - y = 2$ $x + y = 0$ $x + y = 3$

EXTRA. (4 points) A region R is bounded by $x = 0, y = 2x, y = x + 4$ and $x = 3$. Write the iterated integral for the area of R using the order $dx\ dy$.

Problem	Max Points	Points
1	8	
2	6	
3	10	
4	12	
5	9	
6	9	
7	8	
8	8	
Extra	4	
Total		