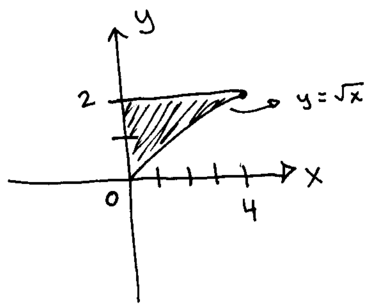
1. (8 points) Consider the double integral. $\int_0^4 \int_{\sqrt{x}}^2 \frac{1}{y^3+1}\, dy\, dx$

(a) (2 points) Why is the current order of integration not desirable?

$\frac{1}{y^3+1}$ is difficult to integrate in terms of y

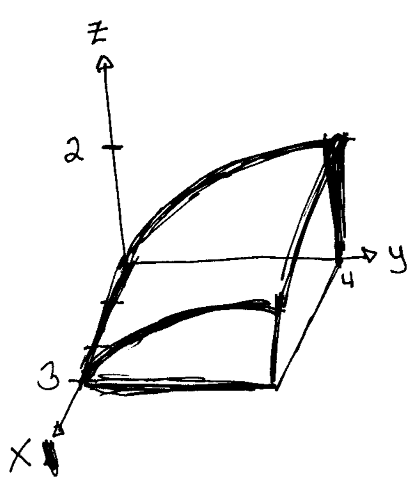
(b) (6 points) Change the order of integration. DO NOT EVALUATE.



$$\int_0^2 \int_0^{y^2} \frac{1}{y^3+1}\, dx\, dy$$

2. (6 points) Change the order of integration for the triple integral using the given order. DO NOT EVALUATE.

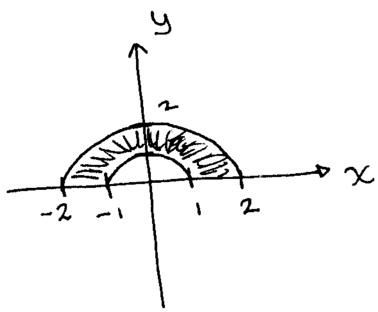
$$\int_0^3 \int_0^4 \int_0^{\sqrt{y}} f(x,y,z)\, dz\, dy\, dx \quad \text{use order } dy\, dx\, dz$$



$$\int_0^2 \int_0^3 \int_{z^2}^4 f(x,y,z)\, dy\, dx\, dz$$

3. (10 points) Evaluate the given integral by first changing to polar coordinates. (**NO** trigonometric substitutions needed.)

$$\int\int_R x+y\, dA \qquad R \text{ is the region such that } \underline{y \geq 0} \text{ and is between the circles } \begin{matrix} x^2+y^2=1 \\ x^2+y^2=4 \end{matrix}$$



$x = r\cos\theta$

$y = r\sin\theta$

$$\int_0^{\pi}\int_1^2 (r\cos\theta + r\sin\theta)\, r \; dr\, d\theta$$

$$= \int_0^{\pi}\int_1^2 (\cos\theta + \sin\theta)\, r^2 \, dr\, d\theta$$

$$= \int_0^{\pi} (\cos\theta + \sin\theta)\, \tfrac{1}{3} r^3 \Big|_1^2 \, d\theta$$

$$= \int_0^{\pi} (\cos\theta + \sin\theta) \tfrac{1}{3}(8-1)\, d\theta$$

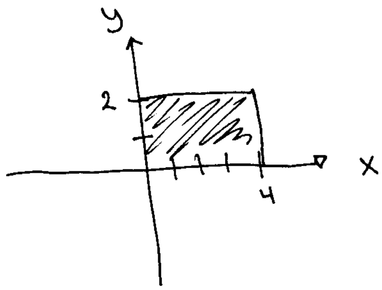
$$= \frac{7}{3}\int_0^{\pi} \cos\theta + \sin\theta \, d\theta$$

$$= \frac{7}{3}\left(\sin\theta - \cos\theta\right)\Big|_0^{\pi}$$

$$= \frac{7}{3}\left[(\sin\pi - \cos\pi) - (\sin 0 - \cos 0)\right]$$

$$= \frac{7}{3}\left[0 - (-1) - 0 + 1\right] = \boxed{\frac{14}{3}}$$

4. (12 points) A thin plate bounded by $x = 0, x = 4, y = 0,$ and $y = 2$ has a density of $\rho(x, y) = 1 + x$. Find its center of mass.



Note that the plate is symmetric about $y = 1$. Also, $\rho(x,y)$ is symmetric about $y=1$ (its independent of y).

So, we have $\bar{y} = 1$

$$\bar{x} = \frac{M_y}{m}$$

$$m = \int_0^2 \int_0^4 1 + x \, dx\, dy$$

$$= \int_0^2 x + \frac{1}{2}x^2 \Big|_0^4 dy = \int_0^2 4 + \frac{16}{2}\, dy = 12 \int_0^2 dy = 12y \Big|_0^2$$

$$= 12 \cdot 2 = \boxed{24}$$

$$M_y = \int_0^2 \int_0^4 (1+x)\, x \, dx\, dy$$

$$= \int_0^2 \int_0^4 x + x^2 \, dx\, dy$$

$$= \int_0^2 \frac{1}{2}x^2 + \frac{1}{3}x^3 \Big|_0^4 dy$$

$$= \frac{16}{2} + \frac{64}{3} \int_0^2 dy$$

$$= \frac{88}{3}(y)\Big|_0^2 = \frac{176}{3}$$

$$\bar{x} = \frac{176}{3} \cdot \frac{1}{24}$$

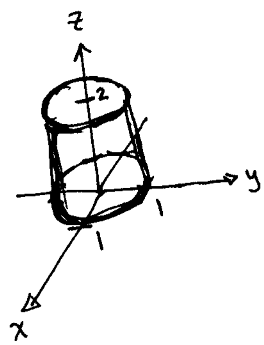
$$= \frac{22}{9} \text{ or } \frac{176}{72}$$

So the center of mass

$$(\bar{x}, \bar{y}) = \left(\frac{176}{72}, 1\right)$$

$$= \left(\frac{22}{9}, 1\right)$$

5. (9 points) Write the integral below using cylindrical coordinates. Note that the solid we are integrating over is a cylinder with height 2. DO NOT EVALUATE.



$$\int_0^2 \int_{-1}^1 \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} (x^2 + y^2 + z^2)\, dy\, dx\, dz$$

$r^2 = x^2 + y^2$

$$\int_0^{2\pi} \int_0^1 \int_0^2 (r^2 + z^2)\, r\, dz\, dr\, d\theta$$

6. (9 points) Write the integral below using spherical coordinates. Note that the solid we are integrating over is a sphere of radius 1. DO NOT EVALUATE.

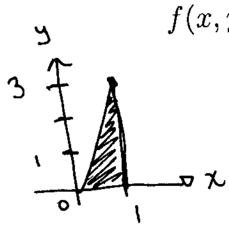
$$\int_{-1}^1 \int_{-\sqrt{1-z^2}}^{\sqrt{1-z^2}} \int_{-\sqrt{1-x^2-z^2}}^{\sqrt{1-x^2-z^2}} (x^2 + y^2 + z^2)^{3/2}\, dy\, dx\, dz$$

$\rho^2 = x^2 + y^2 + z^2$

$$\int_0^{2\pi} \int_0^{\pi} \int_0^1 \rho^2 \cdot \rho^2 \sin\phi\, d\rho\, d\phi\, d\theta$$

7. (8 points) Find the average value of $f(x,y)$ over the region R.

$f(x,y) = xy$ $\quad$ R is the triangle with vertices $(0,0), (1,0)$, and $(1,3)$



Area of triangle is $\frac{3}{2}$

$$\text{average value} = \frac{1}{\frac{3}{2}} \int_0^1 \int_0^3 xy \, dy\,dx$$
$$= \frac{2}{3} \int_0^1 x \cdot \tfrac{1}{2} y^2 \Big|_0^3 \, dx$$
$$= \frac{1}{3} \cdot 9 \int_0^1 x\,dx$$
$$= 3 \cdot \tfrac{1}{2} x^2 \Big|_0^1 = \boxed{\frac{3}{2}}$$

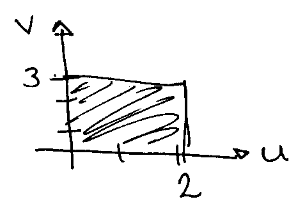
8. (8 points) Make the appropriate change of variables for the integral below. DO NOT EVALUATE.

$$\iint_R (x+y)e^{x^2-y^2}\, dA$$

$e^{(x+y)(x-y)}$

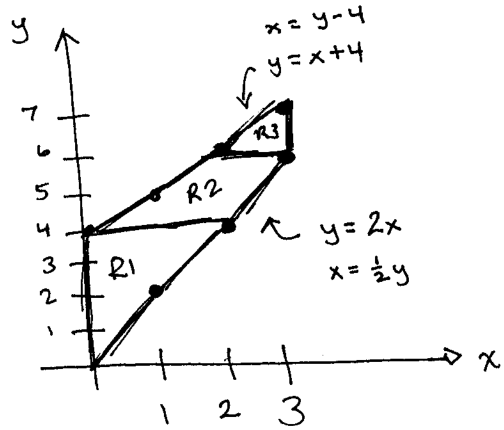
R is the rectangle enclosed by the lines $x-y=0$ $\;$ $x-y=2$ $\;$ $x+y=0$ $\;$ $x+y=3$

let $u = x-y$ $\quad 0 \le u \le 2$
$\quad\;\; v = x+y$ $\quad 0 \le v \le 3$



$$\int_0^2 \int_0^3 v e^{u\cdot v} \, dv\,du$$

EXTRA. (4 points) A region R is bounded by $x = 0, y = 2x, y = x + 4$ and $x = 3$. Write the iterated integral for the area of R using the order $dx\, dy$.



$$\iint_{R1} dA + \iint_{R2} dA + \iint_{R3} dA$$

$$= \int_0^4 \int_0^{\frac{1}{2}y} dx\, dy + \int_4^6 \int_{y-4}^{1/2\, y} dx\, dy + \int_6^7 \int_{x-4}^{3} dx\, dy$$

Problem	Max Points	Points
1	8	
2	6	
3	10	
4	12	
5	9	
6	9	
7	8	
8	8	
Extra	4	
Total		