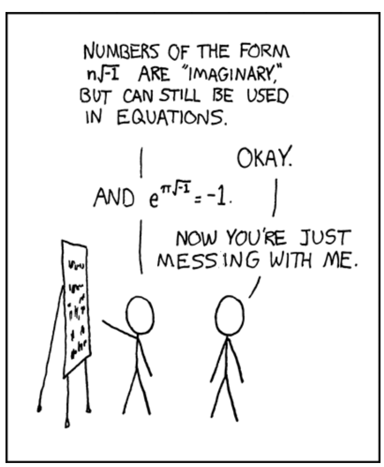
Exam 4

Name: ______________________________

Signature: ______________________________

Student ID: ______________________________

- No notes or books.
- Show all work.
- Good luck!



1. (12 points) Consider the two line integrals.

   A) $\int_C \langle y^2, 2xy \rangle \bullet d\mathbf{r}$ B) $\int_C \langle x^2, 2xy \rangle \bullet d\mathbf{r}$

   (a) (5 points) For which integral does the Fundamental Theorem of Line Integrals apply? Explain.

   (b) (7 points) If the end points of a curve C are $P(2, 4)$ and $Q(1, 0)$, calculate the value of the appropriate line integral.

2. (9 points) Find the potential function for the gradient field

$$\mathbf{F} = \langle yz, xz, xy + 2z \rangle$$

3. (9 points) Parameterize the surface and give the bounds on the parameters.
   Cylinder $y^2 + z^2 = 16$ between $x = 0$ and $x = 5$.

4. (10 points) Answer each

   (a) (5 points) Consider the vector field $\mathbf{F} = \langle xy, 3y^2 \rangle$ on the parameterized curve $C : \mathbf{r}(t) = \langle 11t^4, t^3 \rangle$ for $0 \leq t \leq 1$. Set up the line integral for finding circulation.

   (b) (5 points) Consider $f(x, y) = xy$ on the curve $C : \mathbf{r}(t) = \langle t^2, 2t \rangle$ for $0 \leq t \leq 1$. Calculate the line integral.

5. (14 points) Consider the vector field $\mathbf{F} = \langle \cos y, x^2 \sin y \rangle$ on curve C which is the boundary between the circles $x^2 + y^2 = 1$ and $x^2 + y^2 = 4$.

   (a) (7 points) Using Green's Theorem, set up the integral for circulation.

   (b) (7 points) Using Green's Theorem, set up the integral for flux.

6. (8 points) Use the Divergence Theorem to set up an integral to calculate the flux of $\mathbf{F} = \langle 2x, 2y^3, 2z^3 \rangle$ where surface S is the hemisphere $x^2 + y^2 + z^2 = 9$ where $z \geq 0$.

You may use either formula, but state which formula you want to use.

One parameterization of the curve is:

$$u = \phi \qquad v = \theta$$
$$\mathbf{r}(u, v) = \langle 3 \sin u \cos v, 3 \sin u \sin v, 3 \cos u \rangle$$

7. (8 points) Use Stokes' Theorem to set up an integral to calculate the circulation of $\mathbf{F} = \langle y^2, x, z^2 \rangle$ on the surface S that is part of the plane $2x + y + z = 2$ that lies in the first octant and it oriented upward.

You may use either formula, but state which formula you want to use.

One parameterization of the curve is:

$$x = u \qquad y = v \qquad z = 2 - 2u - v$$
$$\mathbf{r}(u, v) = \langle u, v, 2 - 2u - v \rangle$$

EXTRA. (4 points) For vector field $\mathbf{F} = \langle f, g, h \rangle$ (f, g, h have continuous second partial derivative), prove

$$\nabla \bullet (\nabla \times \mathbf{F}) = 0$$

In words, the divergence of the curl is 0.

Problem	Max Points	Points
1	12	
2	9	
3	9	
4	10	
5	14	
6	8	
7	8	
Extra	4	
Total		