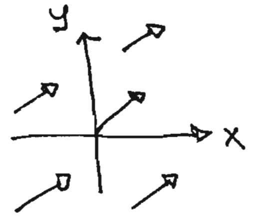
Discussion Questions

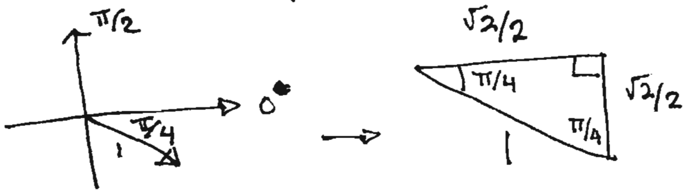
Section 11.1

1) $\vec{v} = 5i - 2j$

2) There are infinitely many vectors of any position vector (and any vector in general) since the tail of the vector can be placed at any point in space.



3) Find unit vector for direction $45°$ south of east.



Since we are in the 4th quadrant
$\vec{v}_u = \langle \frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2} \rangle$

Now multiply by magnitude: $\boxed{\vec{V} = 100 \langle \frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2} \rangle}$

4) $\vec{AB} = \langle 6-3, 10-4 \rangle = \underline{\langle 3, 6 \rangle}$

$\vec{CD} = \langle b+4-(a+2), a-2-(b+5) \rangle = \cancel{b-a}$

$= \underline{\langle b-a+2, a-b-7 \rangle}$

→ we want the components of the vectors to be equal.

x-com. y-com

$\vec{AB} = \vec{CD}$ ⇒ $3 = b-a+2$ and $6 = a-b-7$

$1 = b-a$ $\quad$ $13 = a-b$

→ add equations together

$$\begin{aligned} 1 &= b-a \\ + \; 13 &= a-b \\ \hline 14 &= 0 \end{aligned}$$

→ False!

$\boxed{\text{There is no solution}}$

Discussion Questions

Section 11.2

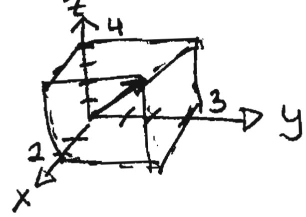
1. $\{ (x,y,z) \in \mathbb{R}^3 \mid x = 2, y = -5, \text{ or } z = 0 \}$

or

All points (x,y,z) in $\mathbb{R}^3$ such that $x = 2, y = -5,$ and/or $z = 0$

2. We want to find the length of the position vector

$$\vec{v} = \langle 2,3,4 \rangle$$



$$|\vec{v}| = \sqrt{2^2+3^2+4^2}$$
$$= \boxed{\sqrt{29} \text{ ft}}$$

Section 11.3

1. (a) Yes, as long as the vectors are in the same space

(b) This is asking for the length of a scalar. The length of a scalar is itself. This expression makes sense.

(c) Does not make sense. You cannot have the dot product of a scalar and a vector.

(d) Yes, this makes sense. The expression is essentially the multiplication of two scalars.

(e) No, you cannot subtract (or add) a ~~scalar~~ vector from a scalar

2. We can find an equation involving a and b.

$$\langle 1,a,b \rangle \cdot \langle 4,-8,2 \rangle = 0$$

$= 0$ because we want them to be $\perp$

$$4 + (-8)a + 2b = 0$$
$$\underline{b = 4a - 2}$$

So we want vectors of the form $\langle 1, a, 4a-2 \rangle$ where $a \in \mathbb{R}$

Discussion Questions

Section 11.4

1. The cross product of two parallel vectors is $\vec{0}$ so the magnitude is $\boxed{0}$
2. use (1) $|u|\cdot|v|\sin\theta$ if you have the angle between them

   (2) $|u\times v|$ or just use the cross product vector itself and use the magnitude equation.

Section 11.5

1. The equation produces vectors
2. Apply the theorem of continuity for each component of $\vec{r}(t)$. If any of the components ~~are~~ is discontinuous at $t=a$, the entire vector is discontinuous at $t=a$.

3. True.

$$\langle 0,0,0\rangle = \langle 3,-1,4\rangle + t\langle 6,-2,8\rangle$$

$$-\langle 3,-1,4\rangle = t\langle 6,-2,8\rangle$$

$$-3 = t\cdot 6 \qquad 1 = -2t \qquad -4 = 8t$$

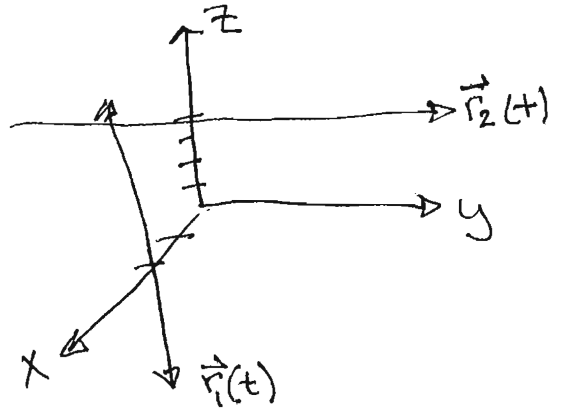
$$-\tfrac{1}{2} = t \qquad -\tfrac{1}{2} = t \qquad -\tfrac{4}{8} = t$$

$$\boxed{\to \text{at } t=-1/2,\ \vec{r}(-1/2) = \langle 0,0,0\rangle}$$

4. False. Consider the lines $\vec{r}_1(t) = \langle 2,0,t\rangle$

   $\vec{r}_2(t) = \langle 0,t,4\rangle$

these lines are not parallel and they do not intersect.



Discussion Questions

Section 11.6

1. a) $\vec{r}'$ points in the direction of the curve
   b) For a specific point p, $\vec{r}'(t)$ gives the rate of change of $\vec{r}$

2. $\vec{r} = \langle \sqrt{t}, 1, t \rangle$

   $\vec{r}' = \langle \frac{1}{2}(t)^{-1/2}, 0, 1 \rangle$

   $\vec{r} \cdot \vec{r}' = 0$

   $\langle \sqrt{t}, 1, t \rangle \cdot \langle \frac{1}{2}(t)^{-1/2}, 0, 1 \rangle = 0$

   $\frac{1}{2}\frac{\sqrt{t}}{\sqrt{t}} + 1(0) + t(1) = 0$

   $\frac{1}{2} + t = 0 \rightarrow \boxed{t = -1/2}$

   orthogonal when $t = -1/2$

Section 11.7

1. The position and velocity vectors are orthogonal to each other at all points along the circle.

2. False. Consider this counter-example: $\vec{v} = \langle \cos t, \sin t \rangle$

3. False. Assuming the initial position is the origin, we can use the general equation for range:

   $$\frac{|v_0|^2 \sin(2\alpha)}{g}$$

   (note: you $\boxed{\text{don't}}$ have to memorize this equation for the exam)

   if $v_0 \rightarrow 2v_0$, then the range is

   $$\frac{|2v_0|^2 \sin(2\alpha)}{g} = \frac{4|v_0|^2 \sin(2\alpha)}{g}$$

   $$\boxed{\begin{aligned} |2\vec{v}_0|^2 &= (2v_1)^2 + (2v_2)^2 + (2v_3)^2 \\ &= 4v_1^2 + 4v_2^2 + 4v_3^2 \\ &= 4(v_1^2 + v_2^2 + v_3^2) \\ &= 4|\vec{v}_0|^2 \end{aligned}}$$

   The range quadruples

Discussion Questions

Section 11.8

1. This is arc length: $\int_a^b \sqrt{(f')^2 + (g')^2 + (h')^2}\, dt$

Section 11.9

1. We can calculate the arc length of a curve using the parameter t directly rather than using the arc length equation for each new b (assuming we always start at t=a)

2. True. $\vec{T}$ points in the direction the curve turns

   $\vec{N}$ points towards the inside part of the curve.

3. False. If friction is present for a car traveling on a flat plane or if the car is on a circular path (cage of death for car?)

Proof Questions

1. $\boxed{\text{Let } u = \vec{u},\ v = \vec{v}}$

$$(u+v)\cdot(u+v) = (u+v)\cdot u + (u+v)\cdot v$$
$$= u\cdot u + v\cdot u + u\cdot v + v\cdot v$$
$$= |u|^2 + u\cdot v + u\cdot v + |v|^2$$

$|u|^2 = u\cdot u$

$u\cdot v = v\cdot u$

$$= |u|^2 + 2u\cdot v + |v|^2 \quad \blacksquare$$

2. $$\vec{u}\times\vec{u} = \begin{vmatrix} i & j & k \\ u_1 & u_2 & u_3 \\ u_1 & u_2 & u_3 \end{vmatrix} = \begin{vmatrix} u_2 & u_3 \\ u_2 & u_3 \end{vmatrix}\hat{i} - \begin{vmatrix} u_1 & u_3 \\ u_1 & u_3 \end{vmatrix}\hat{j} + \begin{vmatrix} u_1 & u_2 \\ u_1 & u_2 \end{vmatrix}\hat{k}$$
$$= (u_2u_3 - u_2u_3)\hat{i} - (u_1u_3 - u_1u_3)\hat{j} + (u_1u_2 - u_1u_2)\hat{k}$$
$$= 0\hat{i} + 0\hat{j} + 0\hat{k}$$
$$= \langle 0,0,0\rangle \quad \blacksquare$$

Discussion Questions

Proof Questions

3. $\vec{r} = \langle at, bt, ct \rangle$

$\vec{r}' = \langle a, b, c \rangle$

Can use dot product. $\vec{r} \cdot \vec{r}' = |\vec{r}||\vec{r}'| \cos\theta$

$$\langle at, bt, ct \rangle \cdot \langle a, b, c \rangle = |\vec{r}||\vec{r}'| \cos\theta$$

$$a^2t + b^2t + c^2t = \sqrt{a^2t^2 + b^2t^2 + c^2t^2}\ \sqrt{a^2+b^2+c^2}\ \cos\theta$$

$$t(a^2+b^2+c^2) = t\ (a^2+b^2+c^2)\ \cos\theta$$

$$1 = \cos\theta$$

$\cos^{-1}(1) = \theta$ $\leftarrow$ θ is always $0°$ or 2π (these are the same)

no matter the value of t ∎

4. $y = f(x)$ let $x = t$, then $y = f(t)$

$\vec{r} = \langle t, f(t) \rangle$

$\vec{r}' = \langle 1, f'(t) \rangle$

Arc length: $L = \displaystyle\int_a^b \sqrt{(1)^2 + [f'(t)]^2}\ dt$