American River College
MATH 402: Calculus III
Quiz 4: 12.4 - 12.6

Name: _____
Student ID: _____
Signature: _____

- Show all work
- No notes, books, or calculators allowed.

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1. (3 points) Find f_x and f_y for the function $f(x,y) = \frac{1}{x}\ln y$

$$f_x = -\frac{1}{x^2}\cdot \ln y$$

$$f_y = \frac{1}{x}\cdot\frac{1}{y}$$

2. (4 points) Find $\frac{dz}{dt}$ for $z = \cos(x+4y)$, where $x(t) = 5t^4$ and $y(t) = \frac{1}{t}$

$$\frac{dz}{dt} = \frac{\partial z}{\partial x}\frac{dx}{dt} + \frac{\partial z}{\partial y}\cdot\frac{dy}{dt} = -20t^3\sin(x+4y) + \frac{4}{t^2}\sin(x+4y)$$

$$= \boxed{\left(-20t^3 + \frac{4}{t^2}\right)\sin\left(5t^4 + \frac{4}{t}\right)}$$

$$\frac{\partial z}{\partial x} = -\sin(x+4y)$$

$$\frac{\partial z}{\partial y} = -\sin(x+4y)\cdot 4$$

$$\frac{dx}{dt} = 20t^3 \qquad \frac{dy}{dt} = -\frac{1}{t^2}$$

3. (3 points) Find the directional derivative of $f(x,y) = \sin(2x+3y)$ at the point $(-6,4)$ in the direction of the unit vector $\langle \frac{\sqrt{3}}{2}, -\frac{1}{2}\rangle$.

$$2(-6) + 3(4) = 0$$

$$\nabla f = \langle 2\cos(2x+3y),\ 3\cos(2x+3y)\rangle$$

$$\nabla f(-6,4) = \langle 2\cos(0),\ 3\cos(0)\rangle = \langle 2, 3\rangle$$

Directional Derivative: $\langle 2,3\rangle \cdot \langle \frac{\sqrt{3}}{2}, -\frac{1}{2}\rangle$

$$= \boxed{\sqrt{3} - \frac{3}{2}}$$