American River College
MATH 402: Calculus III
Quiz 5: 13.1 - 13.3

Name: ____________________
Student ID: ____________________
Signature: ____________________

- Show all work
- No notes, books, or calculators allowed.

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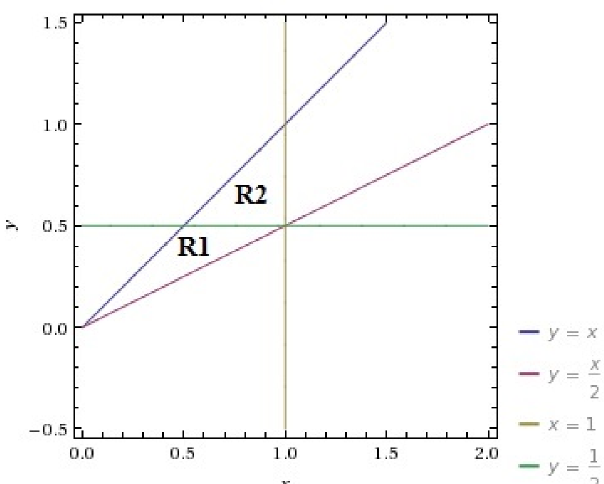
1. (3 points) Evaluate the iterated integral. $\int_0^2 \int_0^1 xy^2 \, dy \, dx$

$$\int_0^2 x \, \frac{1}{3} y^3 \Big|_0^1 dx = \frac{1}{3} \int_0^2 x \, [1-0] \, dx$$
$$= \frac{1}{3} \, \frac{1}{2} x^2 \Big|_0^2$$
$$= \frac{1}{6} \left(2^2 - 0\right) = \frac{2}{3}$$

2. (3 points) Change the order of integration. DO NOT EVALUATE. $\int_0^1 \int_{x/2}^{x} xy \, dy \, dx$

ANSWER: Looking at the figure, we see that there are actually two regions when we use horizontal slices.



For Region 1: $x = 2y$ is the upperbound, $x = y$ is the lower bound. The bounds for y are 0 and 1/2.

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For Region 2: $x = 1$ is the upperbound, $x = y$ is the lower bound. The bounds for y are 1/2 and 1.

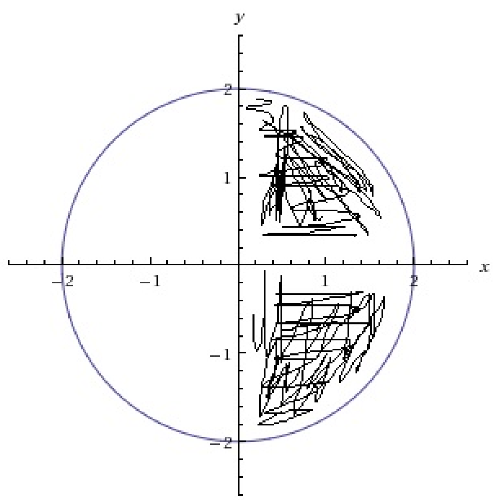
$$\iint_{R1} xy\ dx\ dy + \iint_{R2} xy\ dx\ dy$$

$$\int_0^{1/2}\int_y^{2y} xy\ dx\ dy + \int_{1/2}^{1}\int_y^{1} xy\ dx\ dy$$

3. (4 points) Change the variables from rectangular to polar. DO NOT EVALUATE.

$$\iint \frac{1}{4 + x^2 + y^2}\ dA \qquad R = \left\{(x, y) : x^2 + y^2 \leq 16, x \geq 0\right\}$$

Note that we are working on the half circle:



$$\int_{-\pi/2}^{\pi/2}\int_0^{4} \frac{1}{4 + r^2} \cdot r\ dr\ d\theta$$