American River College
MATH 402: Calculus III
Quiz 7: 14.1 - 14.4

Name: ______________________
Student ID: ______________________
Signature: ______________________

- Show all work
- No notes, books, or calculators allowed.

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1. (3 points) Find the gradient field for the given potential function.

$$\phi(x, y, z) = x^2y + yz$$

$\vec{F} = \nabla\phi = \langle 2xy, x^2 + z, y \rangle$

2. (2 points) Determine if the given vector field is conservative or not.

$$\mathbf{F} = \langle \underbrace{y\sin(xy)}_{\phi_x}, \underbrace{x\sin(xy)}_{\phi_y} \rangle$$

Does $\phi_{xy} = \phi_{yx}$ ?

$\phi_{xy} = \sin(xy) + xy\cos(xy)$

$\phi_{yx} = \sin(xy) + xy\cos(xy)$

$\phi_{xy} = \phi_{yx}$ is true, $\therefore$ the gradient vector field is conservative

3. (5 points) Given parameterized curve $C : \mathbf{v} = \langle 2\cos t, 2\sin t \rangle$, calculate the circulation of the curve in vector field $\mathbf{F} = \langle 2y, 0 \rangle$. Do NOT use Green's Theorem. Hint: use $\sin^2 t = \frac{1}{2}(1 - \cos 2t)$

should be $\vec{r}$, sorry!

$\vec{F} = \langle 2(2\sin t), 0 \rangle = \langle 4\sin t, 0 \rangle$

$\vec{r}' = \langle -2\sin t, 2\cos t \rangle$

Bounds on $t$ are $0 \le t \le 2\pi$ for a circle

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Circulation given by

$$\int_a^b \vec{F} \cdot \vec{r}'\, dt$$

$$= \int_0^{2\pi} \langle 4\sin t, 0 \rangle \cdot \langle -2\sin t, 2\cos t \rangle\, dt$$

$$= \int_0^{2\pi} -8\sin^2 t\, dt$$

$$= -8 \int_0^{2\pi} \tfrac{1}{2}(1 - \cos 2t)\, dt$$

$$= -\frac{8}{2}\left[t - \tfrac{1}{2}\sin(2t)\right]_0^{2\pi}$$

$$= -4\left[(2\pi - 0) - (0 - 0)\right]$$

$$= \boxed{-8\pi}$$