American River College
MATH 402: Calculus III
Quiz 8: 14.4

Name: Key
Student ID: ______________
Signature: ______________

- Show all work
- No notes, books, or calculators allowed.

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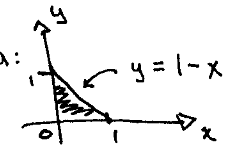
1. (5 points) Find the circulation for $\vec{F} = \langle x^4, xy \rangle$ on the triangular region between the vertices $(0,0)$, $(1,0)$ and $(0,1)$.

$$\text{Curl} = \frac{\partial g}{\partial x} - \frac{\partial f}{\partial y} = y - 0$$

Area:



$$\int_0^1 \int_0^{1-x} y \, dy \, dx$$

$$= \int_0^1 \frac{1}{2}y^2 \Big|_0^{1-x} dx$$

$$= \frac{1}{2}\int_0^1 x^2 - 2x + 1 \, dx$$

$$= \frac{1}{2}\left(\frac{1}{3}x^3 - x^2 + x\right)\Big|_0^1$$

$$= \frac{1}{2}\left(\frac{1}{3} - 1 + x\right) - \frac{1}{2}(0)$$

$$= \boxed{\frac{1}{6}}$$

2. (5 points) Compute the flux of $\vec{F} = \langle 2x + y^3, 3y - x^4 \rangle$ across the unit circle. ← easiest to use polar coordinates for the circle

$$\text{Divergence} = \frac{\partial f}{\partial x} + \frac{\partial g}{\partial y} = 2 + 3 = 5$$

$$\int_0^{2\pi}\int_0^1 5 \, r \, dr \, d\theta$$

$$= 5\int_0^{2\pi} \frac{1}{2}r^2 \Big|_0^1 d\theta$$

$$= \frac{5}{2}\int_0^{2\pi} 1 \, d\theta$$

$$= \frac{5}{2} \cdot 2\pi = \boxed{5\pi}$$

or just do $5 \cdot$ (Area of region) in this case