

Part IV: Hypothesis Testing

Large Samples about Mean

- Case 1.** Average life of “Die Long” batteries **is different** than 60 months. A sample of 64 batteries had an average life of 62 months and st. dev. of 10 months. Let $\alpha = .05$
- Case 2.** Average life of “Die Long” batteries **exceeds** 60 months. A sample of 64 batteries had an average life of 63 months and st. dev. of 10 months. Let $\alpha = .05$
- Case 3.** Average life of “Die Long” batteries **is less than** 60 months. A sample of 64 batteries had an average life of 58 months and st. dev. of 10 months. Let $\alpha = 0.10$

Small Samples about Mean

- Case 4.** Average life of “Die Long” batteries **is different** than 60 months. A sample of 25 batteries had an average life of 62 months and st. dev. of 10 months. Let $\alpha = .05$
- Case 5.** Average life of “Die Long” batteries **exceeds** 60 months. A sample of 254 batteries had an average life of 63 months and st. dev. of 10 months. Let $\alpha = .05$
- Case 6.** Average life of “Die Long” batteries **is less than** 60 months. A sample of 25 batteries had an average life of 58 months and st. dev. of 10 months. Let $\alpha = 0.10$
- Case 7.** Leno Co. claims that the mean life of their batteries is 60 months. Test this claim with $\alpha = 0.02$ if a sample of 6 batteries has a life of 62, 58, 59, 64, 63, 61, months.
- Case 8.** The scores on an aptitude test required for entry into a certain job position have a mean of 500 and a standard deviation of 120. If a random sample of 36 applicants has a mean of 546, is there evidence that their mean score is different from the mean that is expected from all applicants?
- Case 9.** Does an average box of cereal contain more than 368 grams of cereal? A random sample of 36 boxes showed $\bar{X} = 372.5$, and standard deviation is 15 grams. Test at $\alpha = 0.02$ the level.

Proportion

Case 10. At $\alpha = .05$ test that **85%** of stat students pass the course. Out of 200 students only 156 students passed the course.

Case 11. At $\alpha = .10$ test that **more than 85%** of stat students pass the course. Out of 200 students only 172 students passed the course.

Case 10. Prior to election day, an opinion poll among registered voters indicate that 433 voters will vote for incumbent President and 367 will not., Can it be claimed at $\alpha = 0.01$ that incumbent President will win the majority of the votes(getting above 50% of the vote?)

Difference of Two Independent Population Means

Case 11 : Test at the 1% significance level whether the average life of Diehard batteries is longer than Everlast brand. Sample from these two type of batteries are as such:

Die Hard	(μ_1)	$n_1 = 44$	$\bar{x}_1 = 51.8$	$s_1 = 8.5$
Everlast	(μ_2)	$n_2 = 36$	$\bar{x}_2 = 47.4$	$s_2 = 10.7$

Case 12 : A researcher wants to test if the mean GPA of all male and female college students who participate in sports are different. She took a random sample of 33 male students and 38 female students who are involved in sports. She found out the mean GPAs of the two groups to be 2.62 and 2.74, respectively, with the corresponding standard deviations equal to .43 and .38. At 2% significance level, test whether the **mean** GPAs of the two populations **are different**.

Paired Samples

Case 13. A course is intended *to increase* the average sales of salespersons, a random sample of six salespersons and their corresponding sales before and after the course is tabulated as such:

Before	12	18	25	9	14	16	
After	18	24	24	14	19	20	
d=A - B	6	6	-1	5	5	4	$\Sigma d = 25 \quad \bar{d} = 25/6 = 4.17 \quad s_d = 2.64$

At $\alpha = 10\%$, can you conclude that new medication reduces arthritis pain?

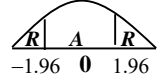
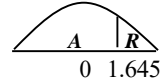
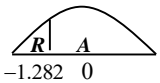
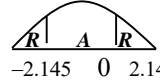
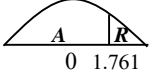
Case 14: A new medication claims that it reduces the pain of arthritis. The following table gives the pain reduction measurement score of eight patients before and after the medication is administrated.

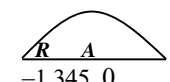
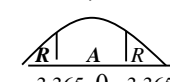
Before	97	72	93	110	78	69	115		
After	72	75	89	91	65	70	90		
d=A - B	-4	3	-4	-19	-13	1	-25	-8	$\Sigma d = -69 \quad \bar{d} = -69/8 = -8.625 \quad s_d = 9.75$

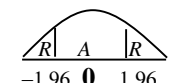
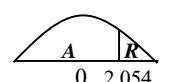
Test the claim at $\alpha = 5\%$

Test of Hypothesis

Worksheets

	Step 1	Step 2	Step 3	Test Statistics (<i>ts</i>)	Conclusion About H_0	Comment About SC	P- value
1	<p>SC $\mu \neq 60$</p> <p>OC $\mu = 60$</p>	<p>$H_0: \mu = 60$</p> <p>$H_1: \mu \neq 60$</p> <p>RTT</p>	<p>$\alpha = .05, n = 64$</p> <p>TTT</p>  <p>CV = ± 1.96</p>	<p>$n = 64, \bar{x} = 62, s = 10$</p> $z = \frac{\sqrt{64}(62-60)}{10} = 1.6$	<p><i>ts</i> falls outside CR then</p> <p>Accept that</p> <p>$H_0: \mu = 60$</p>	<p>Reject that</p> <p>SC: $\mu \neq 60$</p>	<p>0.000585</p> <p>Lower than $\alpha = .10$</p>
2	<p>SC $\mu > 60$</p> <p>OC: $\mu \leq 60$</p>	<p>$H_0: \mu \leq 60$</p> <p>$H_1: \mu > 60$</p> <p>LTT</p>	<p>$\alpha = .05 n = 225$</p> <p>RTT</p>  <p>CV = 1.645</p>	<p>$n = 64, \bar{x} = 63, s = 10$</p> $z = \frac{\sqrt{64}(63-60)}{10} = 2.4$	<p><i>ts</i> falls inside CR then</p> <p>Reject that</p> <p>$H_0: \mu \leq 60$</p>	<p>Accept that</p> <p>SC: $\mu > 60$</p>	<p>0.000233</p> <p>Lower than $\alpha = .05$</p>
3	<p>SC $\mu < 60$</p> <p>OC $\mu \geq 60$</p>	<p>$H_0: \mu \geq 60$</p> <p>$H_1: \mu < 60$</p> <p>TTT</p>	<p>$\alpha = .10 n = 64$</p> <p>LTT</p>  <p>CV = -1.282</p>	<p>$n = 64, \bar{x} = 58, s = 10$</p> $z = \frac{\sqrt{64}(58-60)}{10} = -1.6$	<p><i>ts</i> falls inside CR then</p> <p>Reject that</p> <p>$H_0: \mu \geq 60$</p>	<p>Accept that</p> <p>SC: $\mu < 60$</p>	<p>0.00245</p> <p>Not lower than $\alpha = .01$</p>
4	<p>SC $\mu \neq 60$</p> <p>OC $\mu = 60$</p>	<p>$H_0: \mu = 60$</p> <p>$H_1: \mu \neq 60$</p> <p>TTT</p>	<p>$\alpha = .05, n = 15$</p> <p>TTT</p>  <p>CV = ± 2.145</p>	<p>$n = 15, \bar{x} = 62, s = 10$</p> $t = \frac{\sqrt{15}(62-60)}{10} = 0.775$	<p><i>ts</i> falls outside CR then</p> <p>accept that</p> <p>$H_0: \mu = 60$</p>	<p>Reject that</p> <p>SC: $\mu \neq 60$</p>	<p>0.00779</p> <p>Not lower than $\alpha = .05$</p>
5	<p>SC: $\mu > 60$</p> <p>OC $\mu \leq 60$</p>	<p>$H_0: \mu \leq 60$</p> <p>$H_1: \mu > 60$</p> <p>RTT</p>	<p>$\alpha = .05, n = 15$</p> <p>RTT</p>  <p>CV = 1.761</p>	<p>$n = 15, \bar{x} = 63, s = 10$</p> $t = \frac{\sqrt{15}(63-60)}{10} = 1.162$	<p><i>ts</i> falls outside CR then</p> <p>Accept that</p> <p>$H_0: P \geq .05$</p>	<p>Reject that</p> <p>SC: $P < .05$</p>	<p>0.0014</p> <p>lower than $\alpha = .05$</p>

6	SC: $\mu < 60$ OC: $\mu \geq 60$	H₀: $\mu \geq 60$ H₁: $\mu < 60$ LTT	$\alpha = .10, n = 15$ LTT  -1.345 0 CV = -1.345	$n = 15, \bar{x} = 58, s = 10$ $t = \frac{\sqrt{15}(58 - 60)}{10} = -.775$	ts falls outside CR then Accept that H₀: $\mu \geq 60$	Reject that SC: $\mu < 60$	0.0049 Lower than $\alpha = .01$
7	SC: $\mu = 60$ OC: $\mu \neq 60$	H₀: $\mu = 60$ H₁: $\mu \neq 60$ TTT	$\alpha = .02, n = 6$ TTT  -3.365 0 3.365 CV = ± 3.365	$n = 6, \bar{x} = 61.17, s = 2.32$ $t = \frac{\sqrt{6}(61.17 - 60)}{2.32} = 1.24$	ts falls outside CR then Accept that H₀: $\mu = 60$	Accept that SC: $\mu = 60$	0.16646 Not lower than $\alpha = .05$

	Step 1	Step 2	Step 3	Test Statistics = ts	Conclusion	Comment	P- value
8	SC: $\mu \neq 500$ OC: $\mu = 500$	H₀: $\mu = 500$ H₁: $\mu \neq 500$	$\alpha = .05, n = 36$  -1.96 0 1.96 CV = ± 1.96	$n = 36, \bar{x} = 546, s = 120$ $z = \frac{\sqrt{36}(546 - 500)}{120} = 2.3$	ts falls inside of CR then Reject H₀ : $\mu = 500$	Accept SC: $\mu \neq 500$	
9	SC: $\mu > 368$ OC: $\mu \leq 368$	H₀: $\mu \leq 368$ H₁: $\mu > 368$	$\alpha = 0.02 n = 36$ RTT  0 2.054 CV = 2.054	$n = 36, \bar{x} = 372.5 s = 15$ $z = \frac{\sqrt{36}(372.5 - 368)}{15} = 1.8$	ts falls outside CR then Accept that H₀ : $\mu \leq 368$	Reject that SC: $\mu > 368$	

	Step 1	Step 2	Step 3	Test Statistics = ts	Conclusion	Comment	P -value
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Hypothesis testing

1 A national study shows that in 2006 the average price of gasoline was \$2.51 per gallon. The population standard deviation was shown to be \$0.61. A 2007 random study of 81 Maryland gas stations showed the average price of gasoline per gallon to be \$2.73. Is there significant evidence to suggest that the average price of a gallon of gasoline is more in Maryland in 2007? Test at the 10% significance level.

2 A statewide study shows the average Rocklin SAT score to be 930 with a population standard deviation of 150. A random sample of 225 Roseville SAT scores shows the average SAT score to be 895. Is there significant evidence to suggest that the mean Roseville SAT score is lower than the mean SAT score of Rocklin? Test at the 5% significance level.

3 In 1998, the average price for bananas was 51 cents per pound. In 2003, the following 15 sample prices (in cents) were obtained from local markets: 50 53 55 53 50 57 58 54 48 47 50 57 57 51 55

Is there significant evidence to suggest that the average retail price of bananas is different than 51 cents per pound? Test at the 5% significance level.

4 A 1995 study of 4-year college graduates showed that the average graduate had 132 credit hours. The following 12 samples of credit hours were recorded from Sacramento college students concerning their credit hours upon graduation:

137 135 151 122 131 141 155 132 123 130 145 130

Is there significant evidence to suggest that the average number of credit hours for 4-year graduates is more than 132 credit hours? Test at the 1% significance level.

5 A nationwide study of grocery store goods stated that about 5% of all food products were outdated. At the Weis market in Abingdon, 11 of 515 randomly selected food items were deemed outdated. Is there sufficient evidence to suggest that fewer items are outdated at the Weis Market in Abingdon than the national average? Let us test at the 5% significance level.

6 In 2000, a study of HCC algebra students says that 65% of them pass on their first attempt. In 2004, a survey of 450 algebra students reported that 287 passed on their first attempt. Is there sufficient evidence to suggest that the number of students that passed in 2004 is different than in 2000? Let us test at the 10% significance level.

7 Last year the government made a claim that the average income of the American people was \$33,950. However, a sample of 50 people taken recently showed an average income of \$34,076 with a population standard deviation of \$324. Is the government's estimate too low? Conduct a significance test to see if the true mean is more than the reported average. Use a significance level=0.01.

8. An environmentalist collects a liter of water from 45 different locations along the banks of a stream. He measures the amount of dissolved oxygen in each specimen. The mean oxygen level is 4.62 mg, with the overall standard deviation of 0.92. A water purifying company claims that the mean level of oxygen in the water is 5 mg. Conduct a hypothesis test with $\alpha=0.001$ to determine whether the mean oxygen level is less than 5 mg.

9. A bus company advertised a mean time of 150 minutes for a trip between two cities. A consumer group had reason to believe that the mean time was more than 150 minutes. A sample of 40 trips showed a mean $\bar{x} = 153$ minutes and a standard deviation $s = 7.5$ minutes. At the .05 level of significance, test the consumer group's belief.

10. Data are the total weights of garbage discarded by households in one week (based on data collected as part of the Garbage Project at the University of Arizona). At the 0.01 level of significance, test the claim of the city of Providence supervisor that the mean weight of all garbage discarded by households each week is less than 35 lb., the amount that can be handled by the town. Based on the result, is there any cause for concern that there might be too much garbage to handle?

11. A car sales manager records that in June 45 cars were sold with an average price of \$16,500 and a standard deviation of \$2,000. In December 40 cars were sold with an average price of \$17,300 and a standard deviation of \$1,600. The two populations are normally distributed and the standard deviations are unknown but equal. Is there significant evidence to suggest that the average price of cars in June is less than the average price of cars in December? Test at the 5% significance level.

12. A company claims that its 12-week program significantly reduces weight. The following 4 samples were taken showing the before/after weights of 4 randomly selected customers:

Before	180	195	177	221
After	183	94	80	39
Diff				

Is there significant evidence to suggest that the weights of the customers is significantly less than when they began the 12-week program? Test at the 1% significance level.

13 – Multinomial, Goodness of Fit Test or Sample Proportions follow Population Proportions

A recent study shows the proportions people answered to the question "Where does most of my stress come from at work?"

Demand of Job - 54% Co-Workers – 20% Boss – 10% Layoff - 8% Other reasons – 8%

A study of 800 random employees was asking the same question. The following were the results:

Demand	Co-workers	Boss	Layoff	Other
404	183	94	80	39

Is there significant evidence to suggest that this distribution is different than the population study? Test at the 1% significance level.

15 - **Goodness of Fit Test** Sean Bollins of Bel Air, a local beverage establishment, wants to know if they should change the way they are scheduling their employees. They are currently scheduling the same number of hours on each day of the week. The following represents the number of patrons in a particular week:

Monday	Tuesday	Wednesday	Thursday	Friday
92	71	65	83	89

Is there significant evidence to suggest that the number of patrons is not equal for each day of the week and that they should modify the schedule? Test at the 5% significance level.