

## Binomial Probability

A multiple choice test contains 20 questions. Each question has four or five choices for the correct answer. Only one of the choices is correct. With random guessing, does this test have a binomial probability distribution?

1. A die is tossed 3 times. What is the probability of

- (a) No fives turning up?      (b) 1 five?      (c) 3 fives?

2. Hospital records show that of patients suffering from a certain disease, 75% die of it. What is the probability that of 6 randomly selected patients, 4 will recover?

3. In the old days, there was a probability of 0.8 of success in any attempt to make a telephone call.

Calculate the probability of having 7 successes in 10 attempts

4. A (blindfolded) marksman finds that on the average he hits the target 4 times out of 5. If he fires 4 shots, what is the probability of

- (a) more than 2 hits?      (b) at least 3 misses?

5. A multiple choice test contains 20 questions. Each question has five choices for the correct answer. Only one of the choices is correct. What is the probability of making an 80 with random guessing?

6) A study indicates that 4% of American teenagers have tattoos. You randomly sample 30 teenagers. What is the likelihood that exactly 3 will have a tattoo?

7. A manufacturer of metal pistons finds that on the average, 12% of his pistons are rejected because they are either oversize or undersize. What is the probability that a batch of 10 pistons will contain

- a) no more than 2 rejects?      b) at least 2 rejects?

8. Suppose a die is tossed 5 times. What is the probability of getting exactly 2 fours?

9. Find the mean for the number of sixes that appear when rolling 30 dice.

10. Knowing that about 12% of people are left handed,

a) find the probability of having five left-handed students in a class of twenty five.

b) How many are expected to be left handed?

11. Find the mean for the number of corrected answers on a 20 multiple choice questions (5 choices), if all answers were guessed.

12) A company owns 400 laptops. Each laptop has an 8% probability of not working. You randomly select 20 laptops for your salespeople.

(a) What is the likelihood that 5 will be broken?      (b) What is the likelihood that they will all work?

**(c) What is the likelihood that they will all be broken?**

13) An XYZ cell phone is made from 55 components. Each component has a .002 probability of being defective. What is the probability that an XYZ cell phone will not work perfectly?

14) The ABC Company manufactures toy robots. About 1 toy robot per 100 does not work. You purchase 35 ABC toy robots. What is the probability that exactly 4 do not work?

15) The LMB Company manufactures tires. They claim that only .007 of LMB tires are defective. What is the probability of finding 2 defective tires in a random sample of 50 LMB tires?

16) An HDTV is made from 100 components. Each component has a .005 probability of being defective. What is the probability that an HDTV will not work perfectly?

17. The ratio of boys to girls at birth in Singapore is quite high at 1.09:1.

What proportion of Singapore families with exactly 6 children will have at least 3 boys? (Ignore the probability of multiple births.)

**[Interesting and disturbing trivia:** In most countries the ratio of boys to girls is about 1.04:1, but in China it is 1.15:1.

## Answers

1. A die is tossed 3 times. What is the probability of

a) No fives turning up?  $P(X = 0) = {}^3C_0 (1/6)^0 (5/6)^{3-0} = 125/216 = .5787 = 57.87\%$

(b) Here,  $x = 1$ .  $P(X = 1) = {}^3C_1 (1/6)^1 (5/6)^{3-1} = 75/216 = .34722 = 34.72\%$

(c) Here,  $x = 3$ .  $P(X = 3) = {}^3C_3 (1/6)^3 (5/6)^{3-3} = 1/216 = .004629 = 0.463\%$

2. This is a **binomial** distribution because there are only 2 outcomes (the patient dies, or does not). Let  $X =$  number who recover.

Here,  $n = 6$  and  $x = 4$ . Let  $p = 0.25$  (success - i.e. they live),  $q = 0.75$  (failure, i.e. they die).

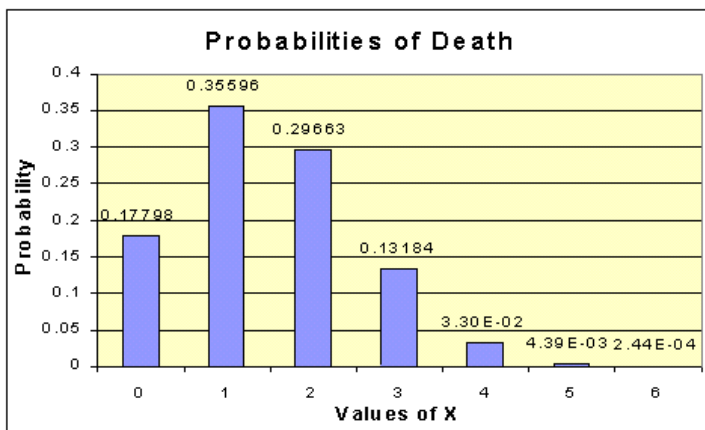
The probability that 4 will recover:  $P(X = 4) = {}^6C_4 (1/4)^4 (3/4)^{6-4} = 0.0329595 = 3.2959\%$

### Histogram of this distribution:

We could calculate all the probabilities involved and we would get:

$X$	Probability
0	0.17798
1	0.35596
2	0.29663
3	0.13184
4	$3.2959 \times 10^{-2}$
5	$4.3945 \times 10^{-3}$
6	$2.4414 \times 10^{-4}$

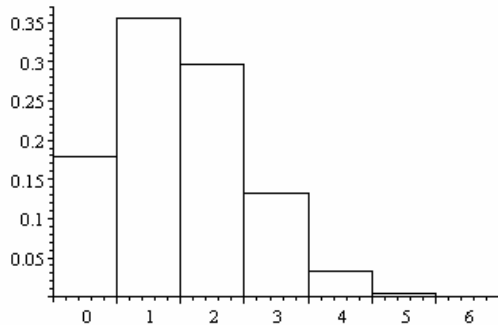
The histogram (using Excel) is as follows:



It means that out of the 6 patients chosen, the probability that none of them will recover is 0.17798, the probability that one will recover is 0.35596, and the probability that all 6 will recover is extremely small.

## SNB "Histogram"

Alternatively, we can use [Scientific Notebook's](#) "Plot Approximate Integral" to give us something approaching the histogram of this experiment. Of course, the  $x$ -values are not quite right in the SNB answer (because it was not designed to do this), so I have made an adjustment to the  $x$ -axis.

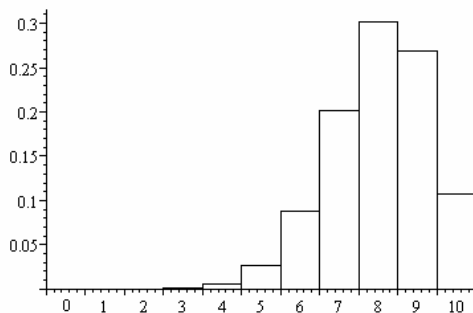


3. Probability of success  $p = 0.8$ , so  $q = 0.2$ .  $X =$  success in getting through.

Probability of 7 successes in 10 attempts:

$$\text{For } X = 7 \text{ then } P(X = 7) = {}^{10}C_7 (0.8)^7 (0.2)^{10-7} = 0.20133 = 20.133\%$$

**Histogram** Using the following function in SNB, we have:



4. A (blindfolded) marksman finds that on the average he hits the target 4 times out of 5. If he fires 4 shots, what is the probability of

Here,  $n = 4$ ,  $p = 0.8$ ,  $q = 0.2$ . Let  $X =$  number of hits. Let  $x_0 =$  no hits,  $x_1 = 1$  hit,  $x_2 = 2$  hits, etc.

a) For  $X = 3$  and  $X = 4$   $P(X) = P(3) + P(4) = {}^4C_3 (0.8)^3 (0.2)^{4-3} + {}^4C_4 (0.8)^4 (0.2)^{4-4} = 0.8192 = 81.92\%$

(b) 3 misses means 1 hit, and 4 misses means 0 hits.

$$\text{For } X = 1 \text{ and } X = 0 \quad P(X) = P(1) + P(0) = {}^4C_1 (0.8)^1 (0.2)^{4-1} + {}^4C_0 (0.8)^0 (0.2)^{4-0} = 0.0272 = 2.72\%$$

5. For  $X = 16$   $P(X = 16) = {}^{20}C_7 (0.2)^{16} (0.8)^{20-16} = 0.000000013 = 0.0000013\%$

6. For  $X = 3$   $P(X = 3) = {}^{30}C_3 (0.4)^3 (0.96)^{30-3} = 0.863 = 86.3\%$   ${}^{30}C_3$

7. Let  $X$  = number of rejected pistons (In this case, "success" means rejection!) Here,  $n = 10$ ,  $p = 0.12$ ,  $q = 0.88$ .

(a) No rejects . For  $x = 0$   $P(X = 0) = {}_{10}C_0 (0.12)^0 (0.88)^{10-0} = 0.2785 = 27.85\%$

One reject For  $x = 1$   $P(X = 1) = {}_{10}C_1 (0.12)^1 (0.88)^{10-1} = 0.3798 = 37.98\%$

Two rejects For  $x = 2$   $P(X = 2) = {}_{10}C_2 (0.12)^2 (0.88)^{10-2} = 0.2330 = 23.30\%$

So the probability of getting no more than 2 rejects is:

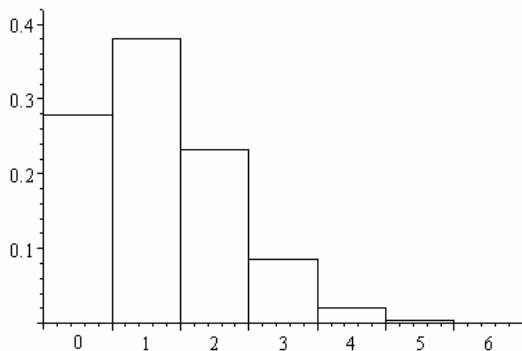
$$\begin{aligned} \text{Probability} &= P(X \leq 2) \\ &= 0.2785 + 0.37977 + 0.23304 \\ &= 0.89131 \end{aligned}$$

(b) We could work out all the cases for  $X = 2, 3, 4, \dots, 10$ , but it is much easier to proceed as follows:

**Probability of at least 2 rejects**

$$\begin{aligned} &= 1 - P(X \leq 1) \\ &= 1 - (P(x_0) + P(x_1)) \\ &= 1 - (0.2785 + 0.37977) \\ &= 0.34173 \end{aligned}$$

Histogram Alternatively, using SNB : for  $X = 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10$



8. This is a binomial experiment in which the number of trials is equal to 5, the number of successes is equal to 2, and the probability of success on a single trial is  $1/6$  or about  $0.167$ . Therefore, the binomial probability is:  $b(2; 5, 0.167) = {}_5C_2 * (0.167)^2 * (0.833)^3$   $b(2; 5, 0.167) = 0.161$

9. Find the mean, for the number of sixes that appear when rolling 30 dice.

Success = "a six is rolled on a single die".  $p = 1/6$ ,  $q = 5/6$ .

The mean is  $30 * (1/6) = 5$ .

**10. a)** For  $X = 5$  then  $P(X = 5) = {}^{25}C_5 (0.12)^5 (0.88)^{25-5} = .1025 = 10.25\%$       **b)**  $np = 25(0.12) = 3$

**11.**  $np = 20(1/5) = 4$

**12. a)**  ${}^{20}C_5 (.08)^5 (.92)^{15} = .0145$       **(b)**  ${}^{20}C_0 (.08)^0 (.92)^{20} = .1887$

**(c)**  ${}^{20}C_{20} (.08)^{20} (.92)^0 = .00000000000000000001$  (note  $^{-22}$  means move the decimal 22 places to the left)

**13.** Probability that it will work (0 defective components)  ${}^{55}C_0 (.002)^0 (.998)^{55} = .896$

Probability that it will not work perfectly is  $1 - .896 = .104$  or 10.4%

**14.**  ${}^{35}C_4 (.01)^4 (.99)^{31} = .00038$

**15.**  ${}^{50}C_2 (.007)^2 (.993)^{48} = .0428$

**16.** Probability that it will work (0 defective components)  ${}^{100}C_0 (.005)^0 (.995)^{100} = .606$

Probability that it will not work perfectly is  $1 - .606 = .394$  or 39.40%

**17.**

The probability of getting a boy is  $\frac{1.09}{1.09 + 1.00} = 0.5215$

Let  $X$  = number of boys in the family. Here,  $n = 6$ ,  $p = 0.5215$ ,  $q = 1 - 0.5215 = 0.4785$

For  $X = 3$  then  $P(X = 3) = {}^6C_3 (0.5215)^3 (0.4785)^{6-3} = .3108 = 31.08\%$

For  $X = 4$  then  $P(X = 4) = {}^6C_4 (0.5215)^4 (0.4785)^{6-4} = .2540 = 25.40\%$

For  $X = 5$  then  $P(X = 5) = {}^6C_5 (0.5215)^5 (0.4785)^{6-5} = .1107 = 11.07\%$

For  $X = 6$  then  $P(X = 6) = {}^6C_6 (0.5215)^6 (0.4785)^{6-6} = .0201 = 2.01\%$

So the probability of getting at least 3 boys is:

$$\begin{aligned} \text{Probability} &= P(X \geq 3) \\ &= 0.31077 + 0.25402 + 0.11074 + 2.0115 \times 10^{-2} \\ &= 0.69565 \end{aligned}$$

**NOTE:** We could have calculated it like this:

$$P(X \geq 3) = 1 - (P(x_0) + P(x_1) + P(x_2))$$